# ECONOMICS OF PLANNING

THEORY AND PRACTICE OF CENTRALLY PLANNED ECONOMIES AND THEIR RELATIONS WITH MARKET ECONOMIES

## HUNGARIAN ISSUE




NO. 3

VOL. 5 · 1965

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## THEORY AND PRACTICE OF CENTRALLY PLANNED ECONOMIES AND THEIR RELATIONS WITH MARKET ECONOMIES

*Published under Scandinavian auspices by the Norwegian Institute of International Affairs in Cooperation with Centre for Russian and East European Studies, University of Birmingham, with financial support from the Swedish Royal Academy of Science, Stockholm, State Council for Social Research, Stockholm, Norwegian Research Council for Science and the Humanities, Oslo, Bank of Finland, Helsinki.*



Annual subscription rate U.S. $ 6.00, postfree. In Scandinavia N. kr. 30.00. 3 issues per year.

Subscription, articles for publication, and books for review should be sent to "Economics of Planning", P. O. box 7030, Oslo. Tel. 60 79 05.



Printed in Norway by A. W. Brøggers Boktrykkeri A/S, Oslo.

**FORMERLY**

**ØST-ØKONOMI**

*The international character and circulation of this journal has necessitated the introduction of the main title "Economics of Planning" to facilitate reference and avoid linguistic difficulties.*

## HUNGARIAN ISSUE

This issue of ECONOMICS OF PLANNING is devoted to application of mathematical methods in Hungarian economic research and planning and has been prepared by the

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ECONOMICS OF PLANNING intends to continue the publication of special issues on planning in different economies

ECONOMICS OF PLANNING
Vol. 5, No. 3, 1965
Printed in Norway

# Mathematical programming as a tool in drawing up the five-year economic plan

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A team of several hundred economists, mathematicians, engineers and practical planners is at present engaged in Hungary in the preparatory work of the first experimental economy-wide programming project.[1] The research work is in direct connection with drawing up the national economic plan for the years 1966–1970; its main purpose is, however, more general than that. We are seeking an answer to the following questions:

– Which are the functions of socialist economic planning and, more specifically, of drawing up the five-year economic plans, that can be carried out be means of mathematical programming methods? What place should be assigned to mathematical programming in the machinery of economic-wide planning?

– Will mathematical programming serve to draw up a more suitable plan than the one based on the usual non-mathematical methods? (The latter will henceforth be called the *traditional* planning methods.) What do the advantages of mathematical programming consist in?

– What are the conditions of the systematical application of mathematical programming methods?

The questions have been repeatedly posed in literature.[2] We considered, however, practical experimenting the best way to obtain truly convincing answers. Mere arguing in favour of the mathematical methods would not do, their usefulness *must be proven.*

It is but natural that our work should show many analogies with the

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[1] Sponsored by the National Planning Board, the research work is carried out under the direction of the author. Of the members of the team, Mrs. László Ábel, Gusztáv Báger, Péter Bod, István Dancs, Mrs. János Deák, György Filep, Tamás Lipták, Béla Martos, András Nagy, Judit Rimler, György Simon, Benedek Schreiber, László Szabó, Márton Tardos, Mrs. László Ujlaki and Tibor Vidos should be specially mentioned for their contribution to the construction of the model and to the calculations carried out up to now.

[2] See, first of all, *L. V. Kantorovich* [5].

application of mathematical programming in other fields. Its essential characteristic, on the other hand, lies in the fact that we have endeavoured to employ the method under socialist conditions and on the national level, as a tool for drawing up the five-year plan and as an integral part of actual practical planning work.

## SOME GENERAL REQUIREMENTS

In the construction of our model we endeavoured to meet the following requirements.

1) We set ourselves the task to construct a computable model. Accordingly, we have employed a linear programming model. We are fully aware of the fact that by using non-continuous variables in our model beside the continuous ones, by representing certain relationships by non-linear equations, by carrying out stochastic programming etc., a truer reflecting of economic reality could be achieved. Such methods of greater precision have already been used before in investigations on a smaller scale.[1] However, for the purposes of the first economy-wide programming project we thought it preferable to revert to the most elementary type of programming model easiest to manage – and even so we had to surmount extreme difficulties as regards computing techniques.

2) Our model has been constructed in a way as to make it in several respects *conform to the structure and the index system of traditional planning*. In doing so we had two aims in mind. First, to be able to meet the greatest possible part of the project's immense data requirements by drawing on data compiled for the purposes of traditional planning. In addition, we should like our results to be, as far as possible, directly comparable with the plan targets determined by traditional methods. (The latter will in the sequel be called the *official program.*) In its relation to traditional planning our research work shows thus a peculiarly dual character: on the one hand, it relies on the official program, on the other hand, it competes with it.

The second requirement – that of conformity to the structure of traditional planning – means at the same time that we make our mathematical model to *simulate* to a certain extent the course and the patterns followed up to now in setting up the five-year plans. This may prove useful in working out a theory of socialist planning.

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[1] Thus, e. g., the national programming project was preceded by a sectoral programming project embracing the entire domain of the man-made fibres industry. Here, a non-linear cost function expressing the advantages of large-scale production was used, uncertain data were treated as random variables etc. (For particulars see [7].)

3) Our calculations must be sufficiently *detailed* to yield utilizable information not only for the central work carried out at the National Planning Board but also for planning at the ministry level. Extreme aggregation in the sector breakdown of the model must be avoided and planning must be possible within the project framework for the most important investments, productive and foreign-trading activities.

Unfortunately, the latter requirements – together with the first one of computability – has compelled us to make certain concessions: we had to renounce to making the model express the time for schedule of the activities. The main argument for the less aggregate but non-dynamical model as against the more aggregate but multi-period model was that traditional planning, too, belonged to the former category. Long-term planning for 15–20 years is still in the elementary stage in this country and the preparation of the five-year plans is not as yet connected in any organized form with working out long-term plans to cover 15 to 20 years' periods. Had we, therefore, constructed a programming model covering two or three five-year-plan periods, we would not have been able to meet the second requirement mentioned above; the data compiled for the purpose of traditional planning would not have provided an adequate basis for our work, nor would it have been possible to compare the results obtained with the official program.

We are well aware that the single-phase character of the model constitutes the greatest weakness of our research work, compelling us as it does to make a number of highly simplifying assumptions. It is to be hoped that in a second experimental calculation it will be possible to remedy this deficiency.

## THE CALCULATIONS ON THE SECTOR LEVEL

The national model is composed of 50 *sector models*. In the initial stage of the calculations each sector model has an "individual life" – it lends itself to independent interpretation and constitutes a tool in the service of lower-level planning.

We define a sector as a productive and foreign-trading unit responsible for a definite range of products or services and – in the assumption of our model – obliged to provide the rest of the national economy with the products or services in question. In the actual practice of Hungarian economic administration, management of and planning for production on the one hand and foreign trade on the other are separated from one another; in our mathematical model, however, the two are organically related.

The individual sectors are generally responsible for several – six to ten, in some cases fifteen to twenty – product groups, aggregates composed of a variety of concrete products. (E.g. bricks, or enamel-ware, or TV receiver sets etc.) Certain sectors have but a single type of output: the electric energy sector's output consists of electric energy, that of the railway sector consists of conveyance etc. In the following, both the product aggregates and the services will be called *products*.

With the concept of the product thus defined, our economy-wide model contains a total of some 400 products. The major part of the variables figuring in the model and representing economic activities is connected with some product (e.g. the establishment of a plant to produce the product in question, its production itself, or its import or export etc.).

The variables representing the economic activities are divided into two main groups: *capital transformation variables* and *operation variables.*

The capital transformation variables represent the economic activities which result in transforming a certain part of the capital stock, of the production and turnover funds, from a definite state in 1966 into a definite state in 1970. Les us give a few examples.

– The 1966 state of a definite productive, transport or service capacity is preserved on the unchanged technological level until 1970. The transformation will in such cases require inputs of a maintenance character.

– A definite industrial plant, power station, railway line etc. is in 1966 already in existence. This will be transformed in the course of the five-year plan period in a definite way and brought into a terminal state which is different from the initial one. (E.g. the plant's old machinery will be partially replaced by modern ones; the railway line will be provided with an automatic safety equipment; some branch of plant cultivation will be further mechanized etc.) Here, the activities aimed at the preservation and the transformation of the 1966 state will be closely interrelated.

– An entirely new plant, transport or service capacity will be brought into existence. In this case we are dealing with a transformation which creates an 1970 capacity in the place of a "zero capital stock" in 1966.

In these examples we spoke of production funds and the transformation of productive capital stocks. The treatment of the country's stock of foreign assets and liabilities is to a certain extent analogous. The credit-raising and debt-repayment activities figuring in the model will increase or decrease the terminal 1970 state of assets and liabilities in the money markets abroad as compared to the initial state in 1966.

The common purpose of all capital transformation variables is to create capacities and possibilities for the economic operation in 1970. Let us now give a few examples also of the operation variables.

– The production of a definite product in 1970. Under this heading are classed all productive activities, including those which will contribute only indirectly to the production of the 400 products of our model. (E.g. the variables representing the production of various semi-finished products which will not leave the plant in this form, as well as the compounding of such products in the same plant in some of the chemical sectors etc.)

– The export and import of definite products in 1970 in definite market relations.

– The collection of interest accrued on credits granted abroad and the payment of interest of foreign debts in 1970.

From what has been said it will be clear that *the program obtained by means of the model constitutes a complex investment, technical development, production, international financial, export and import plan.*

Our model contains a great number of constraints. The first main group of the constraints limits the capital transformation activities, particularly under three aspects:

– From the side of the initial state: e. g. the activities to preserve the initial state are limited by the stocks existing in 1966.

– From the side of the terminal state: e.g. the surplus capacity which can be created by means of the technical reconstruction of an old plant is technically given.

– From the side of the inputs required for the transformation: investment resources are considered limited. Within these, and in accordance with the conventions of traditional planning, separate limits are set to the quotas of domestic, socialist import, and capitalist import machinery available for investment purposes; to the quota of constructions etc. Separate limits are also imposed on the utilization of certain specific types of machinery and construction activity.

Another main group of constraints limits the 1970 activities, primarily under the following aspects:

– Technological equations: the regulation of technical relations between the raw materials, semi-finished and finished products.

– The obligation to satisfy final domestic demand. The non-productive requirements of the population and public institutions are taken as given.

– Foreign trade constraints. The upper bounds representing the marketing difficulties in exports; export obligations undertaken under international agreements as lower bounds; the representation of certain "tie-up" deals such as e.g. export obligations to set off purchases of scarce materials and commodities; global balances of payments and trade; and so on.

– The constraints of labour available. First of all, a non-specified global constraint of the labour force is prescribed. (But this is meant only as the upper constraint; the total utilization of the potential supply of labour will not be regarded as compulsory, because the non-utilized manpower might also be utilized to bring about an increase in leisure time.) In certain domains – such as the supply of engineers or skilled labour, or in research capacity etc. – the available "intellectual capital" is considered as specially limited. An upper bound is also set to the wage fund; this is needed to prevent spending capacity to exceed the supply of commodities and services envisaged when determining final domestic demand.

– The scarce natural resources. These include, first of all, the area of cultivable land, with the different grades of soil quality. Geological resources also come under this heading.

Finally, the third main group of constraints includes those regulating the relationship between capital transformation variables on the one hand and operation variables on the other. Whenever this relationship is unequivocally determined, capital transformation and the operation in 1970 are represented by a common variable. Let us assume e.g. that with a capacity already in operation in 1966 and to be preserved on an unchanged technological level, a single commodity can only be produced. Then, the same variable will represent in the model the preservation between 1966 and 1970 and the operation in 1970. In other cases, however, it will be expedient to connect the two types of activity by means of constraints. E.g. the same productive capacity will produce a variety of products; then, the constraint will prescribe that the capacity requirements of the alternative productive activities for 1970 must not exceed the capacity brought into being by that deadline.

In the sector-level calculations various types of objective function are employed parallelly. (E.g. the minimization of costs in 1970; the maximization of the surplus of the 1970 balance of payments etc.) The sector models are used to carry out a number of sensitivity tests and parametric programming calculations, and the experiences thus gained have served as the basis for our proposals submitted to the management of the sector concerned.

### THE LINKING UP OF THE INDIVIDUAL SECTOR MODELS

In reality, the sector models are not autonomous but related to one another by numerous links. From this point of view the constraint constants of the sector models may be classed into two main groups, viz. *intrasectoral* and *intersectoral* constraints.

The intrasectoral constraints regulate the "internal affairs" of the sector. They include those of the technological equations which describe the flow of products within the sector; the constraints of the sector's initial capital stock, initial capacities etc.; the individual export marketing constraints relating to the sector's products.

The intersectoral constraints, on the other hand, regulate the sector's "external affairs". They include all equations which describe the flow of products between the individual sectors. (E.g. electric energy constitutes the output of the electric energy sector and an input for all other sectors – the balance of electric energy must therefore be considered an intersectoral constraint for all sectors.) The same applies to the allocation of the resources which are being drawn upon by several sectors (e.g. the gross investment quota, the wage fund etc.).

Let us now introduce the following notation:

$A_i$ = matrix of the coefficients figuring in the intersectoral constraints of the $i$-th sector. (In our model $i=1, 2, \ldots, 50$, but for the sake of a more general formulation we will speak of $n$ sectors.)

$B_i$ = matrix of the coefficients figuring in the intrasectoral constraints of the $i$-th sector

$u_i$ = vector of the intersectoral constraint constants of the $i$-th sector

$b_0$ = vector of the macroeconomic bounds of the intersectoral constraints

$b_i$ = vector of the intrasectoral constraint constants of the $i$-th sector

$c_i$ = vector of the objective function coefficients of the $i$-th sector

$x_i$ = the program of the $i$-th sector

$x$ = $[x_1, x_2, \ldots, x_n]$ = the national program

$x_i^{off}$ = the official program of the $i$-th sector as worked out on the basis of traditional methods

$x^{off}$ = the official national program

As pointed out before, the calculations are carried out in two phases. In the first phase, each individual sector carries out its own programming project separately; in the second phase the sector models are linked up and combined into a single large-scale economy-wide model. The separation of the two phases is motivated solely by practical considerations. The sectors will not be ready with their respective models at the same time and it is our wish to use the period of waiting for one another to carry out useful calculations. Besides, experience has shown that in the early stages the models will contain many errors the elimination of which requires repeated checking calculations and practical tests. It is considerably easier to carry out these operations of "running in" on the sector models than on the economy-wide model with its very large dimensions.[1]

Let us now consider the *first phase*. First of all, we will determine the intersectoral constraint vector $u_i$ conform to the official sector program $x_i^{off}$:

$$u_i = A_i\, x_i^{off} \tag{1}$$

When constructing the model it should be ensured that the official program satisfies also the intrasectoral constraints:

$$B_i\, x_i^{off} \leqq b_i \tag{2}$$

Should, in exceptional cases, this condition not be fulfilled, we will proceed to correct the official program and will in the sequel consider this corrected program, which satisfies condition (2), as $x_i^{off}$.

Now we will proceed, by means of the electronic computer, to determine program $x_i^*$, which means the solution of the following linear programming problem:

$$A_i\, x_i = u_i \tag{3a}$$
$$B_i\, x_i = b_i \tag{3b}$$
$$x_i \geqq 0 \tag{3c}$$
$$c_i'\, x_i \rightarrow \max! \tag{3d}$$

We would like to avoid the term "optimal" to distinguish program $x_i^*$ obtained in solution of problem (3), as the optimality of this program is rather relative. (It will depend to a considerable extent, among others,

[1] Moreover, the results of the sector-level calculations may also be utilized – as far as computing techniques are concerned – in launching the programming project on the national level.

on intersectoral constraint vector $u_i$.) We prefer to call it the *dominant* sector program, because it generally dominates the official sector program $x_i^{off}$; both $x_i^*$ and $x_i^{off}$ satisfy the conditions (3a) to (3c), and at the same time the dominant program is considerably better than the official one from the point of view of objective function (3d).[1] In the sector-level calculations carried out so far we have generally maximized the surplus of the balance of external trade as expressed in term of dollars. This objective function showed 5 to 15 per cent savings in the dominant sector programs as against the official sector programs.

Let us now describe the *second phase*. Here, the sector models are combined into a single large-scale national model. We are faced with the following linear programming problem:

$$A_1 x_1 + A_2 x_2 + \ldots + A_n x_n = b_0 \tag{4a}$$

$$\begin{aligned} B_1 x_1 \qquad\qquad\qquad\qquad &= b_1 \\ B_2 x_2 \qquad\qquad\quad &= b_2 \\ \ldots \qquad\qquad & \\ B_n x_n &= b_n \end{aligned} \tag{4b}$$

$$x_1 \geqq 0,\ x_2 \geqq 0,\ \ldots,\ x_n \geqq 0 \tag{4c}$$

$$c_1' x_1 + c_2' x_2 + \ldots c_n' x_n \rightarrow \max! \tag{4d}$$

In relating to our computing possibilities the dimensions of problem (4) are enormous; it contains, in fact, several thousand variables. We will, therefore, have to content ourselves with an *approximation* of the program which constitutes the exact solution of problem (4). Before dealing with the method of approximation, it should be pointed out that by the end of the first phase there will be already at least to national programs available, viz.

$$x^{off} = [x_1^{off},\ x_2^{off},\ \ldots,\ x_n^{off}],$$

the official program of the national economy; and

$$\bar{x} = [x_1^*,\ x_2^*,\ \ldots,\ x_n^*],$$

the team of the dominant sector programs obtained as a result of the calculations carried out in the first phase, – as the solution of problems (3). Let $\bar{x}$ be called in the following the national program of the *first approximation.*

The macroeconomic constraint vector of the intersectoral constraints

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[1] For the sake of simplicity, we will disregard the theoretically not entirely impossible case where $x_i^* = x_i^{off}$. In our actual practice up to now this has not yet occurred.

– $b_0$ – can now be determined, in a manner analogous to calculation (1), as follows:

$$b_0 = \sum_{i=1}^{n} A_i x_i^{off} = \sum_{i=1}^{n} u_i \tag{5}$$

The following statements can now be made about the two national programs already known:

1) Both the official national program $x^{off}$ and the national program of the first approximation $\bar{x}$ are feasible, i.e. they satisfy the conditions (4a) to (4c).

2) The national program of the first approximation $\bar{x}$ dominates the official national program: according to objective function (4d), it is more advantageous than the latter.

In the second phase of the calculations the aim is to find a program which is more advantageous, according to objective function (4d) than the one of the first approximation, i.e. which dominates the official program to an even greater extent.

From what has been said it becomes clear that in the two phases of the calculations we are gradually "drifting away" from the official program. In the first phase vector $u_i$, the official intersectoral distribution of the intersectoral constraints as derived from the official sector program in accordance with equation (1), was still considered binding. In the second phase this restriction is already removed. (I.e. $A_i\, x_i$ may be greater or smaller than $u_i$, as the case may be). It is now only the constraint $b_0$ relating to the national economy as a whole that is derived from the official program in accordance with equation (5), while for the distribution of the constraints $b_0$ among the sectors our mathematical model is left free scope.

It is an obvious precondition to constructing problem (4), to linking up the individual sector models, that each intersectoral constraint should be strictly analogously interpreted in each sector model. In some cases this is easy to ensure, in others, however, it involves great difficulties. Let us mention but one typical example. The output sector will generally strive to plan its output or outlet in a considerably more detailed breakdown than the user sector will be able to state its requirements. (E.g. the output of the man-made fibres industry is given in more detailed breakdown in the man-made fibres industrial sector than the man-made fibres input of the textile industry in the textile-industrial sector.) In the economy-wide problem linked up according to (4) it will, therefore, be necessary to insert in the model's corresponding places certain desaggregating constraints and variables. The latter will serve to break up the aggregated requirements of the user sectors for the output sector concerned.

In the course of linking up the sector models, a whole range of other problems will also emerge (thus e.g. in connection with the constraints regulating the output and distribution of the sectors which are producing capital goods etc.) – these can, however, not be dealt with here because of our restricted space.

## TWO-LEVEL PLANNING

The given state of computing techniques in the country does not allow a direct solution of the large-scale programming problem (4) by means of some usually employed algorithm (as, for instance, the simplex method). Instead, we will have to employ one of the so-called *decomposition methods*, making use of the block-diagonal arrangement of the matrices $B_i$.

After theoretical investigations in several directions and practical calculatory tests we have come to the decision to carry out our first experimental calculations on the basis of the *Dantzig-Wolfe* method.[1] Similarly to the other decomposition methods, this a rather lengthy procedure. It has, however, the great advantage that it brings about a *monotone* improvement in the value of the objective function. Thus, we will obtain a workable result even in the case that we are compelled to stop the iteration before attaining an optimum.[2] Moreover, the Dantzig-Wolfe method affords a possibility to profit at the beginning of the economy-wide calculations from the programs worked out in the first phase of sector-level programming, and to improve the value of the objective function

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[1] See [3].

[2] Another method had been worked out originally for the purpose of the economy-wide planning problem by mathematician *Th. Lipták* and the present author (see [7]). The method in question is based on the game-theoretical interpretation of the problem and utilizes the method of fictitious play. The principal advantage of this latter method of fictitious play consists in the fact that here the dimensions of the model will practically not be limited by the computer's *storing capacity*. The intersectoral part of the problem will, as a matter of fact, require no "regular" linear programming but only the carrying out of a set of considerably simpler operations (such as the calculation of arithmetic means etc.). However, the method is not monotonous: while it approaches the optimum the value of the objective function is strongly fluctuating. It is exactly the monotonous character of the convergence that we consider the principal advantage of the Dantzig-Wolfe method.

as against $\bar{x}$, the program of the first approximation, from the first step of the iteration already[1].

The economy-wide planning carried out on the basis for decomposition methods was termed *two-level planning*. (The term had been originally introduced to designate the algorithm using fictitious play; but in our opinion its generalization is wholly justified.) The work of planning is being carried out on two "levels": partly within the sectors themselves and partly at the National Planning Board, the central government agency responsible for and directing the sectors. On both levels, a certain amount of initial information will be available. Moreover, in the course of the planning process, information will flow between the two levels. The information "output" of the Planning Board's calculations will constitute the information "input" of the sector-level calculations and conversely. It is exactly here that the various decomposition methods differ from one another: in what to consider the initial information at the two levels; in the information that is flowing between the two levels; and in the character of the calculation employed in their processing.

The practice of traditional planning also shows a procedure of a similar character. Previous to drawing up the individual five-year plans, the government – or, rather, on the government's behalf the Planning Board – would lay down the so-called "planning methodology", prescribing the various phases of national economic planning; determining the extent to which the central plan figures must be broken up and "planned back" by the various ministries; and so forth. The decomposition methods will determine the algorithms of the complete planning process, algorithms that will ensure the approximation of the program with the maximum objective function value (or, e.g. in the case of the Dantzig-Wolfe method, its attainment in a finite number of steps).

We should like to point out here at least in a few words the more general, economic-cybernetical significance of the two-level planning models.[2] The literature on mathematical economics divides the models giving a general description of the functioning of the economy into two principal classes. One class of models describes a type of economy which is completely decentralized and composed exclusively of elementary

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[1] To employ the conceptual terminology and the notations of Dantzig-Wolfe's paper [3]: From the dominant sector programs $x_i^*$ as well as from the other sector programs obtained in the course of the sector-level sensitivity tests and satisfying conditions (3b) and (3c) there can be generated such vectors $P_{ik}$ as may form the column vectors of the coefficient matrix of the so-called "extremal problem".

[2] See e. g. the papers of *E. Malinvaud* [9] and *J. Waelbroeck* [10].

units (enterprises, consumers etc.). This is the model employed – primarily after *L. Walras* – by the outstanding representatives of the theory of general equilibrium such as *K. J. Arrow* and *G. Debreu.*[1] The other class of models describes an absolutely centralized economy where every economic equation in centrally planned. The foremost representative of this school is *E. Barone* with his well-known paper.[2] Both types are "single-level" models; the former is composed – to use our own terminology – of sectors only, the latter only of a Planning Board. Our own model, on the other hand, is a two-level one, representing as it does the Planning Board and the sectors simultaneously and in their mutual relationships. Here, the *horizontal* relations between the sectors and the *vertical* ones between the centre and the sectors will appear simultaneously.

We should like to express the opinion that the two-level planning model may have helped in providing a realistic mathematical description of a centrally directed system of planned economy where the component lower-level units still enjoy a comparatively significant measure of independence.

## MATHEMATICAL PROGRAMMING's PLACE IN PLANNING

The planning method briefly described here will, of course, not resolve in itself all tasks involved in drawing up the five-year plan. Two tasks should only be mentioned here which have *not* been carried out.

Our model, though comprehensive, is not complete. It covers the most important sectors but not every sector; the most important products but not production as a whole; the major investment projects but not the entire investment activity of society. Other models, more aggregated but such as embrace completely the national economy, will be needed to complement it. Until other, more advanced models are available, the various static and dynamic input–output models appear suitable to serve this purpose.

The most important open problem, unresolved up to the present, is that of the substantiation of the model's data. For the time being we are, in lack of more suitable resources, drawing on the documentation of traditional planning for a major part of the initial data. A substantiation of the most of these data by means of mathematical-statistical methods

[1] See [1] and [4].

[2] See [2].

would be most expedient. In the machinery of traditional planning, however, this is as yet rather exceptional. The research team engaged in the preparation of the programming project of the national economy is also making efforts to advance this problem: Engel curves are being prepared to substantiate the data relating to consumer demand; regression calculations are carried out to investigate the trends of and the interrelations between investment and foreign trade etc. This, however, is not sufficient. The efforts made to employ mathematical programming as a tool in national economic planning will not serve as a substitute for widening the scope of application of the econometric methods and of the mathematical-statistical analysis of economic trends.

Let us assume in the following that in the next few years an advance will be made in this direction and that the data substituted in the mathematical programming model will be better substantiated. What will then the importance of mathematical planning models in national planning consist in?

In the course of traditional planning, the so-called method of plan coordination is used to ensure the equilibrium of the plan. What this means in actual practice is that the Planning Board and the staff of the planning departments in the ministries try to coordinate their self-established plan figures. The coordination of a long-term plan entails hundreds and thousands of proposals and counter-proposals, memoranda and counter-memoranda, minor and major conferences, debates and telephone calls. The plan which will finally appear to most participants in the cooridnation more or less acceptable is shaped in bargaining and in collective discussions between many hundred planners. This intricate process may be regarded as an "exploration", by means of trial and error, of the solution of an immense equation system consisting of several ten-thousand plan figures as unknowns and of the equations which express their mutual relationships. The mathematical programming of the national plan – and especially the method of two-level planning – provides a mathematical formulation of the plan coordination process, of the general coordination of the plan figures, and mechanizes it by means of the electronic computer.

On the basis of the traditional planning methods it will take 2 to 3 years to draw up a five-year plan. During that period several complete plan proposals may be drawn up *one after the other*, based always on the latest information and on the latest instructions received from the governing political bodies. But never so far have several complete plan variants been submitted *at the same time and parallelly* to the decisionmakers in Hungary. And it is here that the significance of mathematical programming lies, in the fact that by means of sensitivity tests and parametric programming it lends itself for drawing up parallelly a whole range of complete national plant variants.

What is more, these plan variants will not be simply feasible (i.e. complying with the model's constraint constants) and realistic plans in the state of equilibrium but will be *efficient* plans at the same time. (It will be remembered that the criterion of an efficient plan is that there exists no other plan superior to it in every respect, only plans which may be more advantageous in one respect and less advantageous in another respect than the efficient plan.)[1]

It is on purpose that we are avoiding here the term of determining an "optimal" plan. And this not only because we might be compelled – as a consequence of computing difficulties – to stop the calculations before having attained the maximum objective function value. Even if we could attain the extreme value of the objective function, our program could be "optimal" in the general sense of the term only if there would exist a "welfare function" which expressed in itself and synthetically the interest of society. We, on our part, doubt the interpretability of such a function and have not endeavoured to construct any synthetic welfare function of this type when constructing our economy-wide programming model. It is our belief that it will be *sufficient to endeavour that the model's system of constraints and objective function should together give a numerical expression of the general aims of economic policy.*

In our experience, the political body which performs the task of central administration in the socialist economy will not be able to give a numerical description *in advance* of its own "preferences". Mathematical programming, on the other hand, lends itself for generating a whole range of complete and efficient national programs which will reflect a variety of economic policies and indicate at the same time the whole system of economic consequences which the alternative economic policies would entail. (E.g. one program may envisage heavy investments and comparatively little increase in the standard of living by 1970; another may lay greater emphasis on living standards and less on investments; a third one may tend to increase leisure time etc.) The leading political bodies will have to study intensively the complete plan variants representing the alternative economic policies in order to be able to reach a well-founded decision. The program variant accepted in the end may then be described as *the program that will efficiently further the*

[1] For a definition of the efficient program see *T. C. Koopmans* [6].

*economic policy laid down by the decision-making political body*. The definition may be less attractive than the term "optimal program" – but it is more realistic and expresses more accurately the relationship between economic policy and planning in the actual practice of the socialist planned economy.

It will be clear from the above that the task of our investigations is not to define a single national program which would then be unequivocally recommended for execution. Our task will be successfully fulfilled on the day when the two-level mathematical model of the national economy is completed and available for economic administration to carry out experiments on all levels: to work out plan variants and to study the possible consequences of their own decisions. The data of the model will be continually replaced in accordance with the latest informations: the activities which are no longer timely will be eliminated and the newly emerging possibilities included. The mathematical programming model should thus become a *permanent* tool of continuous planning.

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