ECONOMICS OF PLANNING

THEORY AND PRACTICE OF CENTRALLY PLANNED ECONOMIES
AND THEIR RELATIONS WITH MARKET ECONOMIES

CONTENTS

NO. 3
VOL. 9 · 1969

ECONOMICS OF PLANNING

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Published under Scandinavian auspices by the Norwegian Institute of International Affairs in Cooperation with Centre for Russian and East European Studies, University of Birmingham, with financial support from the Swedish Royal Academy of Sciences, Stockholm, State Council for Social Research, Stockholm, Norwegian Research Council for Science and the Humanities, Oslo, Bank of Finland, Helsinki.

Annual subscription rate U.S. $ 6.00, postfree. In Scandinavia N. kr. 30.00. 3 issues per year.

Subscription, articles for publication, and books for review should be sent to "Economics of Planning", P.O. box 2585, Oslo 2. Tel. 56 79 70.

Printed in Norway by A. W. Brøggers Boktrykkeri A/S, Oslo.

ECONOMICS OF PLANNING
Vol. 9, No. 3, 1969
Printed in Norway

Man-machine planning[1]

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1. INTRODUCTION

For the solution of large-scale linear programming problems, it may be useful to resort to what have been called decomposition algorithms. A number of methods have been developed in recent years. Experience has shown, however, that slow convergence to the optimum is a common characteristic of all these methods.[3]

In the following, an approximation method for solving decomposable problems is presented.[4] The underlying mathematical concept is not original; the procedure may be considered as a naive heuristic variant of the Dantzig-Wolfe [2] decomposition algorithm (hereafter, the D-W method.) It cannot guarantee that the optimum of the original, undecomposed problem will be reached. It may, however, help to obtain as early as the first iteration programs with a comparatively favorable objective function value, which also lend themselves readily to practical interpretation.

The subject is treated as follows:

In Chapter 2, definitions are given and assumptions presented. Chapter 3 describes the approximation method in general form. Chapter 4 recommends some computational "tricks" to increase the efficiency of the

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[1] I am grateful to I. Dancs, T. Lipták, B. Martos, R. Norton for useful comments.

[2] The author is Professor of Economics at the Institute of Economics, Hungarian Academy of Sciences, Budapest.

[3] No investigations are known to have been carried out thus far with the aim of comparing, on the basis of genuinely representative computation series (i.e., problems of sufficient size and variety of structure), the practical computational efficiency of the various non-decomposition, and decomposition, exact and approximative methods of linear programming.

[4] For a first description see [4].

procedure. In Chapter 5, the characteristics of the method are discussed. In Chapter 6, one of the possible economic-sociological interpretations of the method is presented.

2. DEFINITIONS AND ASSUMPTIONS

2.1 *Two-level structure*

We have a linear programming problem. It is possible without loss of generality to deal only with the case where the system of constraints consists exclusively of inequalities. The problem is of a two-level structure whenever the variables can be arranged in the form presented in problem (1).[1]

$$\mathbf{A}_1\mathbf{x}_1 + \mathbf{A}_2\mathbf{x}_2 + \ldots + \mathbf{A}_n\mathbf{x}_n \leqq \mathbf{b}_0 \qquad (1a)$$

$$\begin{aligned} \mathbf{B}_1\mathbf{x}_1 \qquad\qquad\qquad & \leqq \mathbf{b}_1 \\ \mathbf{B}_2\mathbf{x}_2 \qquad\quad & \leqq \mathbf{b}_2 \\ \ddots \qquad & \quad\vdots \\ \mathbf{B}_n\mathbf{x}_n & \leqq \mathbf{b}_n \end{aligned} \qquad (1b)$$

$$\mathbf{x}_1 \geqq \mathbf{0}, \quad \mathbf{x}_2 \geqq \mathbf{0}, \quad \ldots, \quad \mathbf{x}_n \geqq \mathbf{0} \qquad (1c)$$

$$\mathbf{c}_1'\mathbf{x}_1 + \mathbf{c}_2'\mathbf{x}_2 + \ldots + \mathbf{c}_n'\mathbf{x}_n \rightarrow \max! \qquad (1d)$$

In the following, problem (1) will be called the *large-scale problem.*

To work out the two-level structure, the activity variables of the large-scale problem have been arranged to form n units which will be called *sectors.*[2] The coefficients of the large-scale matrix correspondingly form n sectoral submatrices.

Let $\mathbf{x} = (\mathbf{x}_1', \mathbf{x}_2', \ldots, \mathbf{x}_n')$ denote the program vector of the large-scale problem and $\mathbf{x}^*$ the optimum program.

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[1] *Notation.* Bold face capital letters denote matrices; bold face small letters, vectors; capital letters in italics, sets; and small letters in italics, real numbers. The prime beside the symbol of a vector denotes a row vector. The asterisk is the sign of optimality. Symbol **E** denotes the unit matrix, **1** a summing vector, always in the dimension corresponding to the formula in question. Empty sets are denoted Ø. As we are dealing here exclusively with linear relationships, the number in the exponent position is in each case an upper index.

[2] The present article adheres, as far as possible, to the terminology used in [3], dealing with two-level planning, and introduced in the first experimental economy-wide programming project connected with the drawing-up of the 1966–70 plan. For details of the latter see [6].

The constraints can be divided into two groups. Group (1a) comprises the *central constraints*, in each of which non-zero coefficients may be found among the submatrices of at least two sectors. There are m central constraints. Group (1b) comprises the special *sector constraints* where non-zero coefficients are found exclusively in the submatrix for the sector concerned.

Let X denote the set of feasible programs of the large-scale problem.

First assumption. The set X is bounded and non-empty; $X \neq \emptyset$.

Let us call matrix $\mathbf{U}$ a central constraint allocation:

$$\mathbf{U} = (\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n),$$

where each $\mathbf{u}_i$ $(i = 1, \ldots, n)$ has the same dimension as the vector $\mathbf{b}_0$, which means that the number of its components is m.

In the linear programming problems (1), each central constraints allocation $\mathbf{U}$ defines n sectoral linear programming problems. Let us call the i-th *sector problem* belonging to $\mathbf{U}$ a linear programming problem with variables $\mathbf{x}_i$ and constraints (3)–(5):

$$\mathbf{A}_i \mathbf{x}_i \leqq \mathbf{u}_i, \tag{3}$$

$$\mathbf{B}_i \mathbf{x}_i \leqq \mathbf{b}_i, \tag{4}$$

$$\mathbf{x}_i \geqq \mathbf{0} \tag{5}$$

2.2 *The degrees of feasibility and optimality*

Let us call $(\mathbf{u}_i, \mathbf{b}_i)$-feasible the sector program $\mathbf{x}_i$ which satisfies the condition (3)–(5). Let $X_i (\mathbf{u}_i, \mathbf{b}_i)$ denote the set of these programs.

Let us call *evaluable central constraints allocation* the matrices $\mathbf{U}$ which satisfy the following two conditions:

$$\mathbf{u}_1 + \mathbf{u}_2 + \ldots \mathbf{u}_n = \mathbf{b}_0. \tag{6}$$

$$X_i (\mathbf{u}_i, \mathbf{b}_i) \neq \emptyset \text{ for any } i \ (i = 1, \ldots n) \tag{7}$$

Let U denote the set of evaluable central constraint allocations.[1] From the first assumption, it will be obvious that

$$U \neq \emptyset. \tag{8}$$

A sector program $\mathbf{x}_i$ will be called $(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$-optimal if it constitutes the optimum solution of the following sector problem:

[1] For the propositions and statements relating to the decomposition of the large-scale problem into a two-level one, and to evaluable central constraint allocations, see [3], pp. 144–150.

$$\begin{aligned} \mathbf{A}_i \mathbf{x}_i &\leqq \mathbf{u}_i \\ \mathbf{B}_i \mathbf{x}_i &\leqq \mathbf{b}_i \\ \mathbf{x}_i &\geqq \mathbf{0} \\ \mathbf{g}_i' \mathbf{x}_i &\to \max! \end{aligned} \tag{9}$$

where $\mathbf{g}_i'$ may be any objective-function coefficient vector (which may differ from the $\mathbf{c}_i$ figuring in the large-scale problem).

A sector program $\mathbf{x}_i$ will be called $(\mathbf{b}_i)$-*feasible* if it satisfies the following constraint system:

$$\begin{aligned} \mathbf{B}_i \mathbf{x}_i &\leqq \mathbf{b}_i \\ \mathbf{x}_i &\geqq \mathbf{0}. \end{aligned} \tag{10}$$

Let $\mathbf{x}_i (\mathbf{b}_i)$ denote the set of $(\mathbf{b}_i)$-feasible programs.

A sector program $\mathbf{x}_i$ will be called $(\mathbf{b}_i, \mathbf{g}_i)$-optimal when it constitutes the optimal solution of the following problem:

$$\begin{aligned} \mathbf{B}_i \mathbf{x}_i &\leqq \mathbf{b}_i \\ \mathbf{x}_i &\geqq \mathbf{0} \\ \mathbf{g}_i' \mathbf{x}_i &\to \max! \end{aligned} \tag{11}$$

2.3 *The comparative program*

In order to employ the approximation method efficiently, it is useful to know beforehand one feasible solution to the large-scale problem. This will be called the comparative program and denoted $\mathbf{x}^0 = (\mathbf{x}_i^{0'}, \mathbf{x}_2^{0'}, \ldots, \mathbf{x}_n^{0'})'$. The name indicates that the programs obtained in the course of computation will be compared with the comparative program.[1]

Second assumption. $\mathbf{x}^0 \in X$.

Let us call *comparative central constraint allocation* the matrix $\mathbf{U}^0$ whose i-th column vector is determined in the following way:

$$\mathbf{u}_i^0 = \mathbf{A}_i \mathbf{x}_i^0 \qquad i = 1, \ldots, n \tag{12}$$

From the first and second assumption as well as from proporsition (8) it follows that the comparative central constraint allocation is evaluable, *i.e.*,

$$\mathbf{U}^0 \in U. \tag{13}$$

[1] In the first experimental Hungarian economy-wide programming project, the so-called *official program* – worked out by practical planners on the basis of non-mathematical, traditional planning method, independently of our model, – was used as the comparative program.

assumption. $\mathbf{c}'\,\mathbf{x}^0 < \mathbf{c}'\,\mathbf{x}^*$.

h assumption. $\mathbf{c}'\,\mathbf{x}_i^0 < \mathbf{c}_i'\,\mathbf{x}_i^1$ for any i† where $\mathbf{x}_i^1$ is the optimum .m of the following problem:

$$\begin{aligned} \mathbf{A}_i\,\mathbf{x}_i &\leqq \mathbf{u}_i^0 \\ \mathbf{B}_i\,\mathbf{x}_i &\leqq \mathbf{b}_i \\ \mathbf{x}_i &\geqq \mathbf{0} \\ \mathbf{c}_i'\,\mathbf{x}_i &\rightarrow \max! \end{aligned} \tag{14}$$

Vector $\mathbf{x}_i^1$ will be called the *sector-optimal* program; this is the optimum program computed with the original vector of objective-function coefficients in the case of comparative central constraint allocation.

2.4 *The plan proposal*

Let us call the *plan proposal* of the i-th sector the vector $\mathbf{t}_i$ defined as follows:

$$\mathbf{t}_i = \mathbf{A}_i\,\mathbf{x}_i \qquad \mathbf{x}_i \in X_i\,(\mathbf{b}_i). \tag{15}$$

Let us denote by the real number s_i the *objective function contribution* of the i-th sector:

$$s_i = \mathbf{c}_i'\,\mathbf{x}_i \qquad \mathbf{x}_i \in X_i\,(\mathbf{b}_i). \tag{16}$$

In the following, the plan proposals are given serial numbers in each sector and the same serial number is given to the objective-function contributions belonging to each sector. The upper index beside the symbol indicates the serial number.

Two special plan proposals will be described together with their objective-function contributions: the comparative and the sector-optimal plan proposals.

$$\left.\begin{aligned} \mathbf{t}_i^0 = \mathbf{A}_i\,\mathbf{x}_i^0 = \mathbf{u}_i^0 \qquad & s_i^0 = \mathbf{c}_i'\,\mathbf{x}_i^0 \\ \mathbf{t}_i^1 = \mathbf{A}_i\,\mathbf{x}_i^1 \qquad & s_i^1 = \mathbf{c}_i'\,\mathbf{x}_i^1 \end{aligned}\right\} i = 1, \ldots, n \qquad \begin{aligned} (17) \\ (18) \end{aligned}$$

3. GENERAL DESCRIPTION OF THE PROCEDURE

In the description, double numbering is employed: the first number is that of the *iteration*, the second one that of the *step* within the iteration.

In accordance with the usual interpretation of decompotion methods,

† On the basis of our practical experience it is virtually certain that the fourth assumption will be valied for any i. Up to the present, we have not encountered any comparative program which would have constituted the optimum solution of problem (14).

it is assumed that some of the steps are carried out by the *c*
the others by the *sectors*. In a sequence of steps, information fl
from the center to the sectors and in the reverse direction.[1] Acco
for each step it is indicated whether it has to be carried out in the
or in the sectors. When a transmission of information takes place in
step in question, its direction is indicated.

Some steps of the process require the solution of an exactly formulate mathematical problem. These operations are carried out in practice on the computer. Other steps, on the other hand, have to be carried out by the practical planners without any exact algorithm, in a heuristic-intuitive manner. Accordingly, it is indicated at every step whether the problem in question is an *algorithmic* or an *heuristic* one. In the case of heuristic steps, only their formal contents are now described. Later on, we return to a discussion of the information the planners may rely on when carrying out these steps.

It is assumed that a comparative program $\mathbf{x}_i^0$ is known for every sector.

3.1 *The first iteration*

Step 1.1 (In the sectors; algorithmic).

The value of $\mathbf{u}_i^0$ is determined according to formula (12). Given this information, problem (14) is solved and the sector-optimal program $\mathbf{x}_i^1$ is determined.

On the basis of the comparative and the sector-optimal programs, plan proposals $\mathbf{t}_i^0$ and $\mathbf{t}_i^1$ are determined according to formulae (17) and (18), together with their objective function contribution s_i^0 and s_i^1.

Step 1.2. (In the sectors; heuristic).

The determination of vector pairs $(\mathbf{u}_i^k, \mathbf{g}_i^k)$ $k=2, 3, \ldots, K_i(1)$.

Step 1.3. (In the sectors; algorithmic).

The solution of the following sector problems on the basis of the constraints and objective function determined in Step 2:

$$\begin{aligned} \mathbf{A}_i \mathbf{x}_i &\leqq \mathbf{u}_i^k \\ \mathbf{B}_i \mathbf{x}_i &\leqq \mathbf{b}_i \qquad K=2, 3, \ldots, K_i(1) \qquad (19) \\ \mathbf{x}_i &\geqq \mathbf{0} \\ \mathbf{g}_i^{k\prime} \mathbf{x}_i &\to \max! \end{aligned}$$

[1] If our process were investigated only from the computational point of view, all operations could, of course, be carried out by the same group of planners with the same computer. In that case, the terms "center" and "sector" would refer only to different phases in the organization of the work.

Let $\mathbf{x}_i^k$ denote the optimum program of problem (19). On the basis of this, let us generate plan proposal $\mathbf{t}_i^k$ as well as the objective function contribution s_i^k belonging to it.

Step 1.4. (In the sectors; algorithmic).

On the basis of the results obtained in Steps 1 and 3, let us formulate the following matrix of plan-proposals and vector of objective function contributions:

$$\mathbf{T}_i(1) = (\mathbf{t}_i^0, \mathbf{t}_i^1, \mathbf{t}_i^2, \ldots, \mathbf{t}_i^{K_i(1)} \tag{20}$$

$$s_i'(1) = (s^0, s^1, s^2, \ldots, s^{K_i(1)} \tag{21}$$

Step 1.5 (From the sectors to the center).

The transmitting of matrices $\mathbf{T}_i(1)$ and vectors $s_i'(1)$.

Step 1.6. (In the center; algorithmic).

The following central problem must be solved:[1]

$$\mathbf{T}_1(1)\,\mathbf{y}_1(1) + \mathbf{T}_2(1)\,\mathbf{y}_2(1) + \ldots + \mathbf{T}_n(1)\,\mathbf{y}_n(1) \leqq \mathbf{b}_0 \tag{22a}$$

$$\begin{aligned} \mathbf{1}'\,\mathbf{y}_1(1) \qquad\qquad\qquad &= 1 \\ \mathbf{1}'\,\mathbf{y}_2(1)\ . \qquad &= 1 \qquad (22b) \\ . \\ .\ \mathbf{1}'\ \ \mathbf{y}_n(1) &= 1 \end{aligned}$$

$$\mathbf{y}_1(1) \geqq \mathbf{0},\ \mathbf{y}_2(1) \geqq \mathbf{0},\ \ldots,\ \mathbf{y}_n(1) \geqq \mathbf{0} \tag{22c}$$

$$\mathbf{s}_1'(1)\,\mathbf{y}_1(1) + \mathbf{s}_2'(1)\,\mathbf{y}_2(1) + \ldots + \mathbf{s}_n'(1)\,\mathbf{y}_n(1) \rightarrow \max! \tag{22d}$$

The role of constraints (22a) is analogous with that of constraints (1a) in large-scale problem; accordingly, we will call them here, too, *central constraints*. Constraints (22b) will be called *combination constraints*.

Weights $\mathbf{y}_i(1)$ are the variables of the central problem, vectors composed each of $(1 + K_i(1))$ components:

$$\mathbf{y}(1) = [\mathbf{y}_1'(1), \mathbf{y}_2'(1), \ldots, \mathbf{y}_n'(1)]'$$

Let $\mathbf{y}(1)^*$ denote the optimum weight vector, the solution of the central problem in the first iteration.

Step 1.7. (In the center; algorithmic).

Let us compute $D(1)$, the *additional gain* obtained in the first iteration:

$$D(1) = \sum_{i=1}^{n} s_i'(1)\,\mathbf{y}_i(1)^* - \mathbf{c}'\,\mathbf{x}^0. \tag{23}$$

[1] The structure of the central problem corresponds to the structure of the "extremal problem" in the D–W method. See formulae (5)-(8) in [2].

3.2 *The further iterations*

Let us now turn to iteration 2, 3, ..., z, ..., $Z-1$. In this section, the z-th iteration is described in general form.

The last (Z-th) iteration is described in Section 3.3.

Step z. 1. (In the center; algorithmic).

We establish the degree to which the upper bounds have been exhausted in the central problem solved in the $(z-1)$-th iteration, and determine *constraint utilization vector* $\mathbf{r}(z)$, the h-th component of which will be:

$$r_h(z) = \frac{b_h - w_h(z-1)}{b_h} \qquad h = 1, \ldots, m \tag{24}$$

where $w_h(z-1)$ is the value of the slack variable figuring in the h-th constraint in the optimum solution of the central problem at the $(z-1)$-th iteration.

When $r_h(z) = 1$, the constraints is *tight*. When $r_h(z) < 1$, the constraint is *loose*, and $r_h(z)$ indicates the *degree of looseness.*

Step z. 2. (In the center; heuristic).

The qualitative evaluation of the components of vector $\mathbf{r}(z)$ whose value is less than 1; the qualitative characterization of the *degree of tightness* ("very tight", "somewhat tight", etc.).

Step z. 3. (From the center to the sectors).

The transmitting of the central information obtained in Steps z. 1 and z. 2. *i.e.*, vector $\mathbf{r}(z)$ and the qualitative evaluations of the degree of tightness.

Step z. 4. (In the sectors; heuristic).

Determining, on the basis of central information received in Step z. 3. and of an analysis of sector programming carried out in earlier interations, the new $\mathbf{u}_1^k$, $\mathbf{g}_i^k$) vectors pairs $[k = K_i(z-1)+1, K_i(z-1)+2, \ldots K_i(z)]$.

The vector pairs are determined according to the following four *viewpoints of formulating the plan proposals:*

(A) If the h-th constraint is loose in the central problem, but has been tight in a particular sector in previous iterations, then the corresponding u_h constraints may be increased over the value given in the earlier iterations. When determining the extent of the increase, that constraint's degree of looseness in the central problem should be taken into account.

(B) If the h-th constraint is tight in the central problem, but not very tight in a particular sector, then the corresponding u_h constraint may be decreased relative to the value prescribed in the earlier iterations. When determing the extent of the decrease, the constraint's degree of tightness in the central problem should be taken into account.

(C) If the h-th constraint has proven very tight in both the sector and the center in previous iterations, then the constraint's sectoral value may be increased over the value prescribed in the earlier iterations.

(D) The objective function in the sector may be reformulated as minimization of the input of some tight constraints. It also may be minimization of the joint input of several tight constraints with suitably chosen weights.

As a possible system of weights we may use the shadow prices belonging to the selected tight constraints in the $(z-1)$-th iteration which are given by the optimum dual solution of the central problem.[1]

To determine the new $(\mathbf{u}_i^k, \mathbf{g}_i^k)$ vectors pairs, whose total number will be $[K_i(z)-K_i(z-1)]$, the viewpoints (A)–(D) listed above can be combined in various ways.[2]

Step z. 5. (In the sectors; algorithmic).

Sector problem (19) is solved with the constraints and objective function determined in Step 4. using the optimum programs obtained, we generate the new $\mathbf{t}_i^k$ plan proposals and s_i^k objective function contributions $[k=K_i(z-1)+1, \ldots, K_i(z)]$.

Step z. 6. (From the sectors to the center).

The new plan proposals and objective function contributions are transmitted to the center.

Step z. 7. (In the center; algorithmic).

The *enlarged* central problem is constructed. By the end of Step z. 6., a total of $(1+K_i(z))$ plan proposals concerning the i-th sector will have been submitted to the center. Accordingly, in the enlarged problem, the

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[1] By accounting for all central constraint inputs at shadow prices and deducting this from the original $\mathbf{c}$ vector, we can reach the objective function of the exact D-W method. This question is dealt with in section 4.2.

[2] In this article – for the sake of simplicity – we are dealing exclusively with the case where there are only upper bounds both in the large scale problem and in all sector problems. In actual practice this is not always the case. In the case of lower bounds, Step z. 4. must be modified accordingly. Should there be, for example, a product balance among the central constraints then a lower bound must be prescribed for the producing sector. In such cases viewpoint (B) of formulating the plan proposals should be applied with the modification that the lower bound is raised (whereas in the user sectors the upper bound is decreased in accordance with viewpoint (B)).

weight vector $\mathbf{y}_i(z)$ and the objective function contribution vector $\mathbf{s}_i'(z)$ will contain $[1+K_i(z)]$ components, and the plan proposal matrix $\mathbf{T}_i(z)$ will have the same number of columns.

The enlarged central problem is solved; the optimum program will be $\mathbf{y}(z)^*$.

Step z. 8. (In the center; algorithmic).

We are computing – in a manner analogous with formula (23) – $D(z)$, the gain in the objective function at iteration z over its initial value in the comparative program.

Step z. 9. (In the center; heuristic).

Judgment is passed on the value of $D(z)$. Should it prove unacceptable, the procedure is continued and the $(z+1)$-th iteration carried out.

If it is acceptable, no further iterations are carried out, and we proceed to the *concluding steps.*

3.3 *Concluding the procedure*

The serial number Z is given to the iteration in which the decision is made not to carry out any further iteration. At that point two concluding steps still must be made.

Step Z. 10. (From the center to sectors).

The optimum program $\mathbf{y}_i(Z)^*$ obtained in the 7th step of the Z-th iteration is transmitted to the sectors.

Step Z. 11. (In the sectors: algorithmic).

We determine the *improved sector program* $\mathbf{x}_i(Z)$:

$$\mathbf{x}_i(Z)=\mathbf{X}_i(Z)\,\mathbf{y}_i(Z)^*, \tag{25}$$

where $\mathbf{X}_i(Z)$ is a matrix whose column vectors are all the $[1+K(Z)]$ sector programs $\mathbf{x}_i^k$ which have been computed up to this point and used for generating the plan proposals:

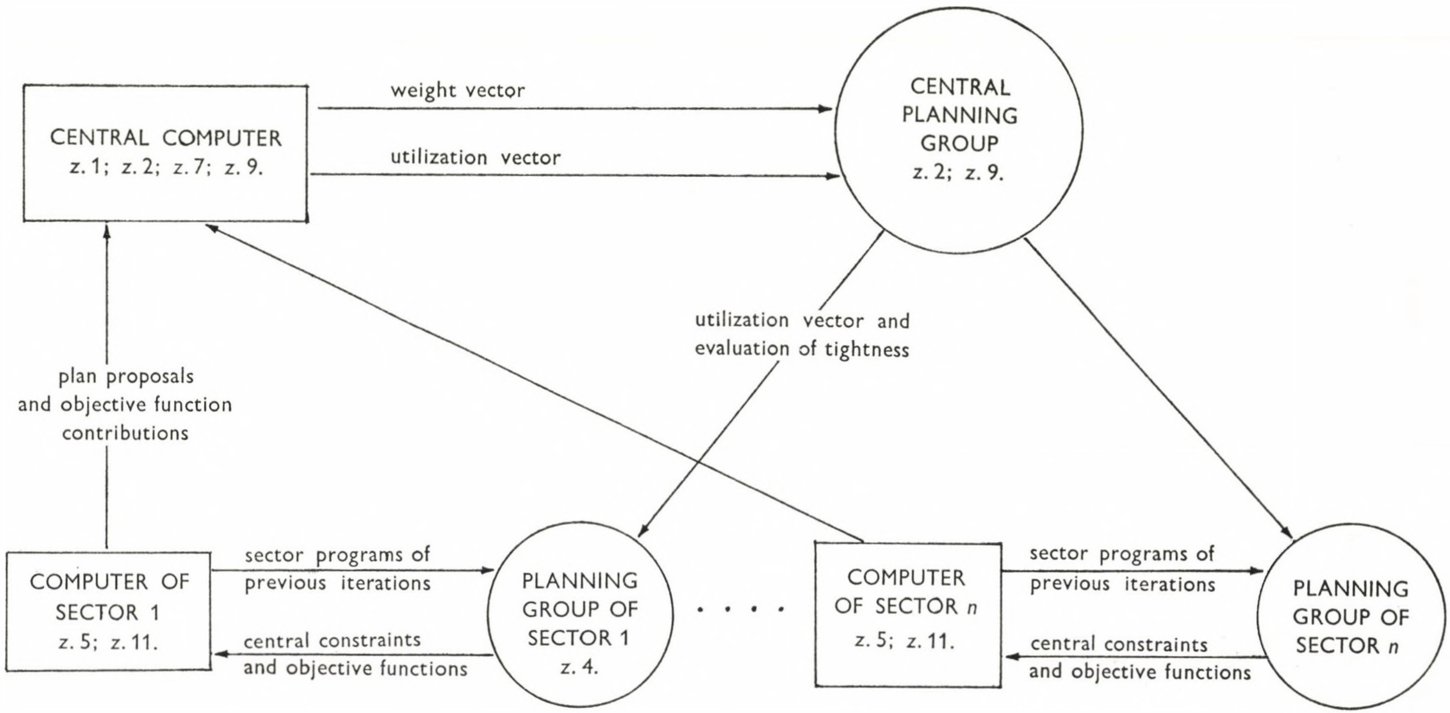
The ensemble of improved sector programs forms the improved program $\mathbf{x}(Z)$:

$$\mathbf{x}(Z)=[\mathbf{x}_1'(Z),\, \mathbf{x}_2'(Z),\, \ldots,\, \mathbf{x}_n'(Z)]' \tag{26}$$

3.4 *The scheme of information flows*

The procedure described in sections 3.1–3.3 is presented schematically below. As in the description above, in this scheme, too, the usual interpretation of decomposition methods is used, namely that some of

SCHEMATIC REPRESENTATION OF STEPS CARRIED OUT IN THE CENTER AND SECTORS



the operations are carried out by the "center" and the others by the "sectors".

Moreover, an institutional interpretation is made: A distinction is drawn between the living planners in both the center and the sectors, the *men* employing the models and methods, on the one hand, and the "dead" *machines* with the data, instructions and algorithms fed into them, on the other. Since the method combines the activities of men and machines, we call it man-machine planning, or the MMP method for short.

4. POSSIBILITIES OF MODIFICATION

4.1 *Some computational "tricks"*

In the following, some possible modifications of the general MMP method which may increase the practical efficiency of the procedure are pointed out.

(1) In the D-W method, the central problem combines exclusively plan proposals for which not $(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$-optimality, but only $(\mathbf{b}_i, \mathbf{g}_i)$-optimality, is required. This is feasible also with the approximation method, provided that there exist already one or two ensembles of plan proposals which are $(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$-optimal with some $\mathbf{U} \in U$.

(2) It is not absolutely necessary to use exclusively $(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$-optimal or $(\mathbf{b}_i, \mathbf{g}_i)$-optimal programs when formulating the plan proposals. These can be supplemented in every sector by some $(\mathbf{u}_i, \mathbf{b}_i)$-feasible and $(\mathbf{b}_i)$-feasible (but not optimal) programs. The latter may be considerably easier to generate than the optimum sector programs.

(3) The approximation to the optimum solution of the large-scale problem may become more difficult computationally when the central problem can combine only $\mathbf{t}_i^k$ vectors with many non-zero components. It may be expedient to enter directly into the central problem as activities some of the original large-scale problems activities which have fewer non-zero components, *e.g.*, those which have only a few non-zero coefficients in the central constraints and only zero coefficients in the special sector constraints.[1]

[1] This was the case with a number of import variables in the course of the economy-wide programming project for 1966–70. These had non-zero coefficients in two central rows only, namely in the product balance concerned and in the balance of foreign exchange. Therefore, several import variables have been built also directly and individually into the central problem, and this made the "blending" of the improved program more flexible.

(4) It will be expedient to avoid equalities in the central problem. Even the constraints for which the economic interpretation requires equality should rather be given as bounds. As a matter of fact, from the viewpoint of the objective function in some cases it may be more advantageous to exceed a bound, *e.g.*, occasionally it may be preferable to produce a surplus rather than to exclude otherwise advantageous plan proposals because they make it impossible to satisfy certain constraints as equalities.

An alternate way of treating this problem is to specify the constraints in question as inequalities and then to assign a negative (positive) coefficient to the corresponding slack variable in the objective function for maximization (minimization) problems. This will tend to enforce equality in those constraints while at the same time avoiding the introduction of artificial variables which considerably retard computational speed. Also, it will permit some plan proposals which are inconsistent with exact equality for those constraints.

(5) Step z. 5. was originally described as one in which the sector defines discrete ($\mathbf{u}_i^k$, $\mathbf{g}_i^k$) vector pairs for the sector programs, providing the basis of the formulation of new plan proposals. In addition (or instead), the methods of parametric programming may also be used. To meet the viewpoints (A), (B) and (C) for sector plan formulation the central vector $\mathbf{u}_i$ allocated to the sector may be given in parametric variants, and to meet viewpoint (D), vector $\mathbf{g}_i$ of the objective function coefficients may be prescribed in parametric form. A single continuous parametric programming computer run may be used for the formulation of a whole series of new plan proposals.

(6) Step z. 4. was originally described as one carried out by the sector independently, using the central information obtained in Step 2.3. The procedure may, however, be completed as follows:

The sectors in every iteration report also the dual solutions of the optimum sector programs used for generating the plan proposals, or, rather, from these dual solutions, the shadow prices of the central constraints. The center will compare these, will prescribe for the next iteration the following:

The upper bound of the h-th central resource must be raised (*i.e.*, viewpoint (B) must be respected) in the sectors where the shadow price belonging to the h-th constraints is high. The bound of the same resource must be decreased (*i.e.*, viewpoint (C) must be respected) in the sectors where the shadow price is low.[1]

[1] The computational "trick" described in paragraph (6) brings the basic concept of the decomposition method of fictitious play into the MMP method. (See [3]). There,

(7) In the course of practical application, usually not just a single computation, but rather a whole computation series, is carried out. The members of the series may differ from each other both in the nature of the objective function and in the value of the individual components of the constraint vector. When applying the MMP method, it is possible to make preparations for this in advance:

– All objective functions to be employed in the series are used in the sector programming computations when determining the ($\mathbf{u}_i$, $\mathbf{b}_i$, $\mathbf{g}_i$)-optimal programs.

– Modification of some central constraints is anticipated in the course of the computation series. For example, when we wish to explore various values of the h-th central constraint, we will work out various sector plan proposals, of which some use more and some use less of the ht-h constraint.

In this way, it will be possible to work out a universal, versatile central problem. This can be employed in the case of any member in the computation series, at least in the first iteration, for computing the initial $\mathbf{y}(1)$ weight vector.

4.2 *Change-over to the exact D-W method*

Our method being a variant of the D-W method we may after any iteration change over from the heuristic approximation method to the exact D-W algorithm. Let us suppose that after the $\check{z}$-th iteration, we decided to continue the computation – on the basis of the results obtained up to that point by means of the approximation method – according to the exact algorithm. Let us write down the $(\check{z}+1)$-th iteration, retaining the numeration of steps as described in section 3.2. We will now omit the indication of individual steps as "algorithmic" because this applies now naturally to each step.

Step $\check{z}+1$). *1.* (In the center).

Read the optimum dual solution of the central problem in the z-th iteration. Let $\mathbf{p}(\check{z})$ denote the vector of shadow prices belonging to central constraints (22a).

Step $(\check{z}+1)$. *2.*

This step is not carried out.

Step $(\check{z}+1)$. *3.* (From center to sectors).

The transmission of vector $\mathbf{p}(\check{z})$.

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the inter-sectoral regrouping of resources is taking place on the basis of the indications of shadow prices obtained in the sectoral programming solutions.

Step $(\check{z}+1)$. *4.* (In the sectors).

Formulate the following sector problem:

$$\begin{aligned} \mathbf{B}_i\,\mathbf{x}_i &\leqq \mathbf{b}_i \\ \mathbf{x}_i &\geqq \mathbf{0} \\ (\mathbf{c}_i' - \mathbf{p}'(\check{z})\,\mathbf{A}_i)\,\mathbf{x}_i &\rightarrow \max! \end{aligned} \tag{27}$$

Step $(\check{z}+1)$. *5.* Solve problem (27), and generate, using the solution, the new plan proposal together with the objective function contribution belonging to it.

Step $(\check{z}+1)$. *6.*–$(\check{z}+1)$. *7.* These correspond to the identical steps of the z-th iteration described in section 3.2.[1]

When applying the MMP method, the computer program can be written in such a manner that the planners may switch from the approximation to the exact method at any iteration. Accordingly, the approximation method may also be interpreted as the preparatory phase of the D-W method which provides a suitable initial program for the exact D-W computation.

5. THE PROPERTIES OF THE MMP METHOD

5.1 *Provable properties*

In the following, those properties of the general method described in Chapter 3 will be dealt with which are formally demonstrable.

First property. (*Feasibility*). The improved programs generated with the general approximation method constitute the feasible solution of the large-scale problem: $\mathbf{x}(Z) \in X$.

Proof. First, it must be proven that there is always a feasible solution to the central problem. This follows trivially from the fact that at least two solutions are known which are *per definitionem* feasible:

$$\mathbf{y}_i = [\mathbf{y}_i^0 = 1,\ \mathbf{y}_i^k = 0\ (k=1, \ldots K_i(Z))] \qquad i=1, \ldots, n \tag{28}$$

or:

$$\mathbf{y}_i = [\mathbf{y}_i^0 = 0,\ \mathbf{y}^1 = 1,\ \mathbf{y}_i^k = 0\ (k=2, \ldots K_i(Z))] \qquad i=1, \ldots, n \tag{29}$$

In the following we must demonstrate that $\mathbf{x}(Z)$ is a feasible solution of the large-scale problem.

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[1] Step 9 may be complemented with an estimation of the distance from the optimum. The formula is known, from the literature on the subject. For its description see, *e.g.*, *J. Stahl's* article [9]. Stahl's estimation formula was used in the D-W computer program worked out in 1966–1967 by the Computing Center of the Hungarian Academy of Sciences.

On the one hand, central constraints (22a) of the central problem ensure the satisfaction of central constraints (1a) of the large-scale problem, since

$$\sum_{i=1}^{n} \mathbf{T}_i(Z)\,\mathbf{y}_i(Z)^* = \sum_{i=1}^{n} \mathbf{A}_i\,\mathbf{x}_i(Z) \leqq \mathbf{b}_0 \tag{30}$$

On the other hand, the plan proposals in the central problem are based exclusively on ($\mathbf{b}_i$)-feasible programs. The combination constraints (22b) ensure that the improved program should be composed of the convex combination of these, *i.e.*, that the special sector constraints (1b) of the large-scale problem should also be satisfied. The plan proposals also are required to observe the non-negativity restrictions on activity levels. Therefore the improved program meets all the constraints of the large-scale problem.

Second property. (*Improvement*). The general MMP method enables us to generate a program, the objective function value of which is definitely higher than that of the comparative program:

$$\mathbf{c}'\,\mathbf{x}(Z) > \mathbf{c}'\,\mathbf{x}^0. \tag{31}$$

Proof. There is certainly known at least one program of which it is obvious that it is more advantageous than the comparative program, and this is the program described in equation (29). This is the optimum program of sector problem (14) for every sector with comparative central constraint allocation $\mathbf{U}^0$.

Third property. (*Monotonicity*). The objective function value obtained in the z-th iteration is not lower than that obtained in the $(z-1)$-th iteration:

$$\mathbf{c}'\,\mathbf{x}(z) \geqq \mathbf{c}'\,\mathbf{x}(z-1). \tag{32}$$

Proof. This follows directly from the fact that – in accordance with Step z. 7 – the plan proposals accumulated up to the $(z-1)$-th iteration are not abandoned in the z-th iteration. The new plan proposal obtained in the z-th iteration will be included only if it improves the value of the objective function.

5.2 *Likely properties*

The MMP method's efficiency will be – provided that it is expertly applied – considerably higher than could be guaranteed on the basis of its mathematically provable properties. Some non-provable but likely properties are described below.

Here, not only the general method described in Chapter 3 but also the possibilities of modification and completion outlined in Chapter 4 will be kept in mind.

To illustrate our point, examples of the method's applications in economics and planning are given. Our arguments can, however, be extended to application in other fields.

Fourth property. (*Realism*). The simplex-type, finite and exact methods of linear programming, with the D-W method among them, proceed through the extreme points of the convex polyhedron forming the set of feasible programs, leaping from extreme point to extreme point. In the course of this, usually we start from absurd programs which do not lend themselves to economic interpretation, with the basis containing only the unit vectors. Then, when the program becomes more interpretable the objective function value is still rather disadvantageous. It is only towards the end of the iteration process that non-absurd programs are reached which are economically interpretable and sufficiently advantageous from the point of view of the objective function; from this point further iterations will then lead up to the optimum.

The proposed MMP approximation method starts from an interior point of the polyhedron[1] and usually also ends at an interior point. But already the interior point reached in the first iteration will be "sufficiently advantageous". This is guaranteed by the first and second properties: the fact that a sound, more or less rational program based on information from outside the model was included from the outset in the plan proposals.

Of course the D-W method may be started from a "realistic" program instead of requiring it to pass through phase I of the simplex method. The MMP method, in fact, may be regarded as a procedure for generating an initial program for the D-W method, if we wish to pursue convergence.

In the further iteration with the MMP method, the program's soundness, rationality and economic interpretability will be enhanced by the fact that the plan proposals are not only $(\mathbf{b}_i)$-feasible, as in the case of the D-W method, but also $(\mathbf{u}_i, \mathbf{b}_i)$-feasible. In addition, since $(\mathbf{u}_i, \mathbf{b}_i)$-feasibility is based largely on an evaluable central constraint allocation, the sector programs as well as the central program will be meaningful in

[1] From this point of view, the efficiency of the MMP method ought to be compared – by means of experimental computations – with that of the *gradient* methods which start also from an interior point.

economic terms. Information from outside the model also may be used to facilitate the determination of a realistic evaluable central allocation.

Fifth property. (*Continuous improvement*). In can be rendered probable, although not proven, that if Step z. 4. is skillfully carried out, the value of the objective function not only will not deteriorate from iteration to iteration but will improve considerably.

This is based on two economic considerations.

(a) Inter-sectoral re-allocation. In Step z. 4. new plan proposals were worked out which highly economize the scarce resources and products (see viewpoints (B) and (D)). In addition, plan proposals also are worked out which use more of those resources and products. (See viewpoint (C).) This affords the possibility of carrying out inter-sectoral re-allocation in the $(z+1)$-th interation. Should it be advantageous from the point of view of the objective function, the plan proposal economizing to a high degree a scarce resource can be included in one of the sectors and the consequent savings can be utilized by the plan proposals of other sectors which require more of the scarce resource.

(b) Substitution among the factors. In Step z. 4. plan proposals also are prepared which use more of the loosely-constrained resources (see viewpoint (A)) and less of the tightly-constrained resources (see viewpoint (B) and (C)). Should it be advantageous from the point of view of the objective function, then the new plan proposals which when put together, carry out substitution among factors will be included in the central program.

Actually, the exact mathematical programming methods, with the exact decomposition methods among them, employ similar economic principles. They do this, however, by observing simultaneously the marginal returns (shadow prices) of *all* resources, factors and products, carrying out small corrections by using all of them simultaneously. (Thus, the D-W method carries out the correction of the evaluation of all central constraints simultaneously in the objective function of the sector computation; the [3] method of fictitious play corrects at the same time all components of the $\mathbf{u}_i$ vector, and so on).

In the case of the MMP approximation method, on the other hand, when working out new plan proposal (in Step z.4.), we will manipulate only *part* of the resources, factors and products by means of constraints adjustments or a change in the objective function, while leaving the rest unchanged.

To delineate the appropriate constraint adjustments the exact decomposition methods can employ only the information brought into the model in advance or the information which is computed in the course of the iterative process itself (*e.g.*, the shadow prices of the central problem in the D-W procedure, or the shadow prices of the sector problems in the [3] algorithm using fictitous play). In the case of the MMP method, on the other hand, the planner is likely to know independently of the model, which resources, factors and products are tight and which ones loose in the large-scale problem, and within that in the individual sectors; it is with these in mind that he can help intersectoral re-allocations and substitutions among the factors. It is partly on the basis of these (and only partly on that of algorithmic central information formed in Step z. 1. of the iteration) that he will decide on the resources, factors and products where constraint adjustment should be carried out in Step z. 4.

Both inter-sectoral re-allocation and substitution among the factors will be facilitated if at least some of the plan proposals are "extremist". For example we have a plan proposal drawn up purposefully to require improbably high investment and at the same time economize highly on manpower; or conversely another plan proposal may use an improbably high amount of manpower but extremely little investment. A number of such programs, which follow clearly differentiated technological or political-economic strategies can be imagined. Such "extremist" programs can easily be generated by choosing suitable ($\mathbf{u}_i$, $\mathbf{b}_i$, $\mathbf{g}_i$)-optimal plan proposals.

It may be expected that the extreme plan proposals will not receive a weight near unity but still will appear with a positive weight in the improved program. Their existence will facilitate "blending" in the central problem solution, in the most flexible manner the more efficient intersectoral allocation, and the best combination of the factors.

When speaking of the fourth property, it was already pointed out that a sufficient number of "sound" plan proposals will be needed which are somewhat similar to the usual allocation. In addition, however, "extremist", or "one-sided", plan proposals generated in the above spirit are also needed, to be able to reach the suitable "blend" in the shortest possible computation time.

Speaking of the improvement of the program, some remarks should be made also concerning the termination of the computation. This is an inevitably arbitrary decision, for which the planner will have to rely again to a high degree on information from outside the model. On the one hand, he will be able, on the basis of his practical knowledge and experiences, to realize whether $D(z)$, *i.e.*, the improvement against the comparative program, is significant or not, taking into account both the absolute value of the improvement, its magnitude relative to the comparative program, and its rate of improvement in each iteration (whether it is slowing or accelerating, etc.).[1] On the other hand, he will take into account the "price" that has to be paid for an iteration. What intellectual and material forces are tied up in carrying out an iteration? Is it worthwhile to engage the capacity of the planners and the computers in a further iteration, or would it be more reasonable to start instead working on a new problem, and carry out the first iterations of a new plan variant?

Without wishing to lay down a general rule, we may venture to say that within the framework of the MMP method it will hardly be worthwhile to carry out more than 5 to 10 iterations. By then, the heuristic ideas of the planners usually will be exhausted. Should we wish to go farther in improving the program, then it would be reasonable to change over to the exact method, as described in section 4.2, taking upon ourselves the cost of the further slow but certain convergence.

Sixth property. (*The interpretability of the plan proposal*). In the original D-W method, the meaning of the optimal program is clear. The plan proposals obtained in the course of the individual iterations have, however, no marked economic meaning, no special characteristic of their own.

In the case of the MMP method, on the other hand, it is endeavoured to give each plan proposal some independent economic characteristic, some marked "profile", *e.g.*, "a program economizing on labor, with a loose investment quota", or "economizing on dollars, to the disadvantage of domestic inputs", etc.

It is in the definition of the "special characteristics" of these proposals that the information material available to the planners from outside the model is expressed, information that plays an important role again in Step 4. As a matter of fact, it is on the basis of this information that it will become clear what modifications in the constraints and what changes in the objective function is it worthwhile to carry out in order to give the plan proposal a definite "profile".

This will, at the same time, ensure that it is not only the final result, the improved program $\mathbf{x}(Z) = \mathbf{X}(Z)\ \mathbf{y}(Z)^*$, that lends itself to analysis. In

[1] It was already mentioned above that the exact D-W method affords the possibility of estimating the distance from the optimum. In our experience, however, the algorithm's formula which sets an upper bound to the improvement which can still be achieved, usually underestimates the improvement already achieved and overestimates that which can still be realized. Accordingly the application of the formula does not promise much benefit; for the practical planner improvement over the comparative program usually will mean more.

each individual iteration, and especially in the last one, the weights y_i^k will also be significant and suitable for direct economic analysis.

On the basis of what has been said above, let us now summarize the stages where the MMP method makes use of information obtained from outside the model:

(a) Including the comparative program in the plan proposals;
(b) in determining the evaluable central constraint allocations;
(c) in determining vector pairs $(\mathbf{u}_i, \mathbf{g}_i)$, which provide a basis for generating new plan proposals, in order to facilitate rational intersectoral regrouping and rational substitution between the factors;
(d) in forming the "profile" of the plan proposals;
(e) in evaluating the additional returns secured as compared with the comparative programs, and in terminating the computation.

One of the tasks of research aimed at further developing the method described in the paper will be to work out new ideas and suggestions to accelerate the procedure of improving the program and to carry out the heuristic steps more efficiently.

5.3 *Practical experience*

The approximation method has been used to carry out some minor experimental computations. In addition it was once applied in practice to a large-size problem within the framework of economy-wide programming for 1966–1970 in Hungary.

The large-scale problem contained a total of 2055 constraints and 2424 activity variables (auxiliary variables excluded). For lack of adequate computational facilities, the exact solution of the problem could not be undertaken. Any attempt to solve the problem either directly, without decomposition, or by employing one of the decomposition methods would have rendered the computation rather slow. This would have tied up to computers for a long time, involved high costs and increased the sources of computational errors, endangering thereby the accuracy of the final result.

Therefore, the approximation method described in Chapters 3 and 4 of this paper has been resorted to. This has enabled the computation of 22 plan variants, *i.e.*, the determination of 22 different improved programs $\mathbf{x}(Z)$ obtained with the application of various vector pairs $(\mathbf{b}_0, \mathbf{c})$.

As quickly as the first iteration, usually programs were obtained in which plan proposals other than the comparative $\mathbf{t}_i^0$ and the sector-optimal $\mathbf{t}_i^1$ ones appeared with a positive weight.

The improved programs showed a considerable improvement in the value of the objective function as compared with the comparative program. Let us give some examples:

– The program maximizing consumption ensures a consumption level 5.2 percent higher than that in the comparative program.

– The program minimizing labor input saves 6 percent of the manpower requirements of the comparative program.[1]

5.4 *The justification of employing approximation methods*

The discussion of the concrete method outlined in this paper provides an opportunity to make some general remarks on the justification of employing approximation methods in solving economic planning problems.

It is certainly not our intention to make a virtue of our computational difficulties. Thus, in the economy-wide programming for 1966–1970, we would have preferred to employ an exact method in solving the concrete problem. It is not here that the real problem lies but in the following dilemma, well known to all model constructors.

Let us assume that the mathematical character of the model has already been decided upon, and confine ourselves to the case discussed here, that of linear programming. In that case it is not the size of the model that will be given beforehand (for a particular country, at a given time, or for some definite research team) but the computational limitations will be given. The storage capacity and speed of the available computers, the utilizable machine time and funds all set a limit to the dimension of the linear programming problem that can be solved with the exact method. The model constructor's dilemma lies in the fact that he must content himself *either* with this size *or* – should he want a larger model – with an approximation instead of an exact method.

In both cases a concession is made to the detriment of accuracy. In the second case, this is obvious. However, the inaccuracy implicit in resorting to a more aggregate model formulation must not be overlooked either. The constructing of economic models is in itself an "approximation method". Every model represents an inaccurate and simplified copy of reality. The more factors left out of consideration, the more the possibility of choice is restricted, the higher the degree of aggregation (*i.e.*, the greater the extent to which things are added together which are not directly additive), the less accurate will be the model, in two senses of the word. On the one hand, a feasible program of the aggregate model

[1] See [6].

may not be feasible in reality because it fails to satisfy a whole range of constraints which are not included in the model but which nevertheless exist in actual fact. On the other hand, an exactly optimal program of the aggregate model may in reality be sub-optimal, because the realistic alternatives which probably would have appeared in the optimal program of a more detailed model had not been included in the variables.

The computations based on the model involve two procedures. First, the infinitely complicated reality is reformulated into a mathematical problem; then, the mathematical problem is solved. It is left to the model constructor to decide in which of the two procedures should he be more accurate, to the detriment of the accuracy of the other procedure.

This is a problem of *general character*, not exclusively related to present-day computational difficulties in Hungary. If we had computers ten times as large as the present ones, the question would again pose itself: Should we content ourselves with the exact solution of the problem which we had formerly been obliged to approach with the approximation method. Or should we make a step forward in model construction, reflecting reality in greater detail in a larger model (*e.g.*, by replacing single-period planning by multi-period dynamic planning) but carrying out the computation again on the basis of some approximation method?

No unequivecal and generally valid solution of this dilemma exist. In practice, it will be best to follow both paths in parallel, *i.e.*, to construct, on the one hand, models with a higher degree of aggreagation for exact computations and, on the other hand, more disaggregative models for approximation methods. This is exactly what we did when experimenting with the mathematical programming methods to be used in Hungarian five-year planning: exact computations were carried out with a linear programming model of a size of about 80×100,[1] and the approximation method was employed in the case of another model of size about 2000×2500. The result obtained with the two models can be compared with each other and used for reciprocal control.

Here, we have reached an even wider problem of mathematical planning, which is dealt with only cursorily in this paper, namely the relationship between the planner and the computer.

In the literature on simulation, the term "man-machine"-simulation is widely applied. It denotes experiments in which some of the operations are carried out by the computer on the basis of an algorithm fixed in advance, while others are improvised by the persons taking part in the experiment, by those who analyse the results obtained in the meantime

[1] See [5].

from the computer. By analogy, in the case of the approximation method we may speak of *"man-machine planning"*, this is what the circles and rectangles connected with each other in the schema in Chapter 3 were intended to illustrate. This is the reason why we call our method the MMP method.

It must be pointed out that this is not the only case where this applies. Cooperation of this type between algorithmic, mechanized operations and heuristic, intuitive and improvised human intellectual activity is highly characteristic of all mathematical planning. Even when applying the exact methods, there will be much intuition, in the construction of the model and in the partly subjective estimation of the data; during the computation, in determining the computation series and sensitivity tests to be carried out; and, finally, in the evaluation and analysis of the results and in actual decision-making.

6. ON THE INTERPRETATION OF THE METHOD – CONFLICT AND COMPROMISE

The algorithms of mathematical programming, and especially the decomposition methods usually can be given some economic interpretation as a formal abstract model of planning and of decision processes. It is a common characteristic of all interpretations that they do not pretend to formalize every essential feature of planning and of the basis for decisions. The various algorithms usually emphasize only one or another element of the process.

MMP method described in this paper can also be given an economic (and even a general, sociological) interpretation. When giving this interpretation, we must, naturally, detach ourselves from the computational aspects of the problem. In this connection, we must not think any longer of the original large-scale problem, the solution of which we want to approximate, nor of the fact that the main purpose of generating plan proposals is to advance the improvement of the objective function belonging to the large-scale problem, etc. The interpretation is the following.

In every organization – be it the state, an administrative unit, some social or political institution, an enterprise, etc. – there exist *internal conflicts*. Various parts, sub-units, interest groups will take a stand on the questions of the day on the basis of their own views, real or supposed interests. Their opinions, suggestions and proposals often will contradict each other. For example, each sub-unit will claim more of the organization's common resources and will want to contribute less. Even within the sub-unit, the various groups will interpret the specific interests of the sub-unit in various ways.

The collective life of the organizations will be possible in spite of these conflicts because some *compromise* will be made between the contradictory proposals. When forming the compromise, various criteria may play a part, according to how the organization's supreme decision-makers assess the common interest.

Modern sociologists and economists have dealt extensively with the problem of conflict and compromise within the organizations, mainly on the basis of empirical observation.[1]

The MMP method – and especially the computation series carried out by means of the MMP method and described in Chapter 4 – may be interpreted as the formalization of the process of working out the conflicting proposals and the compromise made between them.

Conflicts exist on two levels. On the one hand, the plan proposals compete with one another within the basic unit, the i-th sector. These can be regarded – due to the fact that each one of them has some marked "profile" of its own – as the expression of the different views and opinions arising within the sector. Should the sector represent an enterprise, then the plan proposals may reflect the viewpoints of the different groups within the firm.[2] On the other hand, conflicts exist between the sectors, regarding the allocation of the common resources and the carrying of the common burdens and obligations.

It is the central aim of the MMP method to work out a reasonable compromise between the conflicting proposals. As in real life, here too, the compromise will be formed in an iterative process. First, a temporary preliminary compromise will emerge (first iteration). Analysing the weaknesses of this, the decision-makers will ask for further proposals, on the basis of which they will endeavor to reach a more suitable compromise, and so forth.

The fact that we are dealing here with an approximation and not with an exact method, does not weaken this interpretation; in fact, it renders it rather more realistic. Such processes do not progress towards an "opti-

[1] It is primarily the sociologists engaged in the examination of "formal organization" and the representatives of the so-called "behaviorist" school who investigate the problem from this point of view. See [1] and [8].

[2] For example, within an enterprise, the financial department would propose a program aimed at increased profits; the sales department would push the production of goods in demand; the technical development section would urge increased productivity. Within the latter, one group of engineers would recommend technology "A", another group technology "B", and so forth.

mal" compromise based on some strict criterion in actual reality either. Instead one will content oneself with a "second-best", and "acceptable", final solution. Instead of strictly enforcing a single optimality criterion, experiments are carried out on the basis of several different viewpoints – which, in the case of our formalized procedure, corresponds to the fact that a series of computations is carried out with the same universal initial central problem but with varying objective functions.

In our opinion, conflict and compromise constitute a particularly important element in the processes of planning and decision-making. Our MMP method is but one of the possible formalizations; it will be worthwhile to continue research in this direction, to work out and employ also other mathematical models of representing conflicts and compromises.

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