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"MATHEMATICAL PROGRAMMING IN SOCIALIST PLANNING"

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Contents

Preface

The present study is a translation of the articles published in Közgazdasági Szemle No.1 and 2 1968.

We have completed the first experimental computing sequence for the multi-level programming of the national economy, only some complementary and checking computations are still to be done. We should like to publish the various experiences of our work on the domains of economics, methodology of planning and economic policy in detailed studies and later possibly in the form of a book. However, the evaluation of the results and the preparation of publications require much time. So as a first information we publish this report outlining the structure of our model and discussing some problems of the future implementation of our method.

PART I.:

A MULTI-LEVEL PROGRAMMING MODEL OF THE NATIONAL ECONOMY

The idea to link up the various sector models with one another and to unite them in a single economy-wide programming calculation was first put forward in Hungary six years ago. Four years ago, the practical preparations for a national programming project were launched.x/ Since that time, a large team of theoretical economists, practical planners, mathematicians, computing technicians and engineers had been active at the project - at the height of the work their number reached several hundred.xx/ The aim was to put to practical test, on the basis of factual figures, the method of multi-level planning.

This aim was accomplished, though the realization took more time and encountered greater difficulties than had been expected. The five-year economic plan's multi-level planning model, suited to mathematical programming, stands now ready for use. True, it is but a "prototype" which should be further improved to meet the requirements that might arise in the future.

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x/ The basic ideas of combining the various sectoral planning models were outlined in the author's paper "The linking of central and sectoral programming projects" /Budapest, 1961, Computing Centre of the Hungarian Academy of Sciences, in manuscript/. In 1962, another paper on the subject was published in co-authorship with Th. Lipták under the title "Two-level planning" /Budapest, Computing Centre of the Hungarian Academy of Sciences, mimeographed/. (Published in Econometrica 33 /1965/ p. 141-169.) Preparations for practical calculations started in December 1963.

xx/ The project was directed by a central team headed by the author and composed by members of the Computing Centre of the Hungarian Academy of Sciences and of the Institute for Economic Planning of the National Planning Office.

Yet it exists, no longer on the pages of theoretical papers only but filled in with figures, on punched tapes and cards and magnetic tapes that can be fed into the electronic computer, ready for planning calculations. Moreover, the first computation series based on the central model which had resulted from the union of the sector models, has already been successfully carried out.

Some complementary calculations have still remained to be worked out. It may take a couple of years to analyse the lessons to be drawn from long years of research work, lessons that will benefit both economic policy and the theory and methodology of planning. The present paper will be confined to a short description of the model's structure and information basis; the problems of application will be dealt with in another paper. A more detailed and complete analysis will have to wait till later publications.

1. The structure of the model

The method employed represents an application of <u>mathematical programming</u> in economic planning. In the first experimental calculation the most simple form, that of <u>linear</u> programming, was used. In the more remote future, as soon as computing-technical facilities permit, it should be possible to change over to other programming methods /containing also discontinuous variables and non-linear relationships/ which are more adequate from the point of view of economic theory.

Paragraph 1.3. of this paper is to deal with the model's special "multi-level" structure. In the first approach of the description this aspect is still disregarded and the economy-wide model treated as a single large linear programming problem.

1.1. The variables

The model performs the programming for the production, distribution and foreign trade of 491 <u>products</u>. These are generally not concrete and fully specified commodities but rather product groups or aggregates. /E.g. "coal and anthracite", "block aluminium", "boring machines", "bricks", "cotton-type yarns", "canned meat" etc./ When defining the products it was endeavoured to follow the nomenclature of "priority products" used in the planning work of the National Planning Office and the ministries.

The production and foreign trade of the individual products in the last year of the plan period /in the present case, 1970/ is generally represented by seven "standard" variables. These are:

1. The production of the product in the last plan year, with fixed capital that had already been in operation at the beginning of the plan period and remains unchanged up till its end. This requires only the maintenance of the old fixed capital during the plan period.

2. The production of the product in the last plan year, with fixed capital which had already been in operation at the beginning of the plan period but whose technology is changed by reconstruction in the course of that period. This requires, accordingly, not only the maintenance but also the technical reconstruction of the old fixed capital during the plan period.

The value of variables 1. and 2. determines the fate of old fixed capital. If variable 1. has a positive value, the old fixed capital must be maintained without any change; if variable 2. does so, a reconstruction of the old fixed capital must be carried out. If the value of both these variables is zero, the old fixed capital must be dismantled.

3. The production of the product in the last plan year, with fixed capital set up in the course of the plan period.

In the case of numerous products, several variables - such as 3.1., 3.2. etc. - figure instead of a single one, to represent the different technological variants of new plants.

The computation of variables 1., 2. and 3. determines the pattern of production in the last plan year. At the same time, it also determines the pattern of gross investment /maintenance, reconstruction and new investment/ during the plan period. This again involves the elaboration of some basic estimates of <u>technical development</u>, as the proportions of variables 1., 2. and 3. /and also those of variables 3.1., 3.2. etc. representing the technological variants/ will to a great extent determine the technological pattern of production.

4. Imports of the product from socialist markets.

5. Imports of the product from capitalist markets.

6. Exports of the product to socialist markets.

7. Exports of the product to capitalist markets.

As may be seen, the program computes not only the volume of foreign trade but also its breakdown by major markets.

To sum up: the economy-wide program yields a complex production, investment, technical development and foreign trading plan with respect to 491 priority products.

To measure volume, the physical units generally employed in planning were used wherever possible /in the case of 406 out of a total 491 products/. Only in respect of the products which cannot be measured in this way were value terms used, based on producer prices.

Not each of the 491 products has all seven standard variables. In the case of some products the old fixed capital does not lend itself to technical reconstruction. Others again cannot be used for the purposes of all four foreign-trading activities.

The 491 products do not represent the whole of social production. The external sphere, that which is not represented by any variable in the model, fall into two parts. The first of these comprises the sectors not covered by the model. Cases in point are metallurgy and transport.[x] The 1965 input-output table of the National Bureau of Statistics devided the national economy into 74 sectors.[xx] Out of these, only 52 sectors appear with their priority products in the model described here.

The second part of the external sphere comprises the residual, non-priority, activities - if any - of the sectors which appear in the model with their priority products only.

As regards the sectors left out entirely, they could in principle be built into any future model without great difficulty. The treatment of the non-priority activities of the sectors covered by the present model, on the other hand, involves some rather difficult and intricate problems which will be dealt with in the second part of the paper.

When defining the model's variables it was assumed that the individual products have the same homogeneous properties irrespective of whether domestically produced or imported, intended for domestic use or for export. This is an assumption that is universally made in the construction of models and in planning based on product balances, the traditional method adopted in socialist countries.

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x/ In the case of some sectors the fact that they were entirely left out was due solely to practical organizational reasons, namely to the failure to form the research teams which could have worked out the corresponding part of the model. Nor was it endeavoured to cover all branches of the economy in these first experimental computations; the method could be tested without that. Even so, the model became ultimately larger and more comprehensive than intended. As a matter of fact, it was originally planned to cover 10 to 20 sectors only.

xx/ The 74 sectors taken into account in the comparison do not include the three private sectors, the three residual sectors with a "sundry" character, and the handicrafts sector.

When defining the model's variables, only the investment projects not extending beyond the plan period were taken into account. The resources serving the pruposes of so-called "overlapping" investments were deducted from the resources available for allocation by the model.

The model has a total of 2424 activity variables. The figure does not include the so-called auxiliary /slack, redundancy and artificial/ variables.

1.2. The economic content of the constraints

The system of constraints may be considered from two different aspects of classification. Let us first group the constraints according to their economic contents.

The constraints are presented in tabular form on p. 8. The table gives the number of constraints belonging to each constraint group, the unit of measurement used, and the time period/i.e. whether the data presented refer to the last plan year or to the whole plan period/.

1. Internal product balances. These coordinate in the model the outputs represented by variables /production, import/ with the inputs represented by variables /productive input, export, additional consumption/ and the inputs given as constant /"compulsory" private and public consumption, productive input into the external sphere, investment and reconstruction requirements in the last plan year, changes in inventories/.

To most products there belongs an internal product balance. In some cases, however, the balance of input and output is expressed in a somewhat different form. This happens, e.g., when producers use a different breakdown for their output

A survey of the constraints

No.	Constraint group	Unit of measurement	Period	Number of central constraints	Number of main-branch constraints	Number of sectoral constraints	Number of all constraints
1.	Internal product balances	Physical units or forints	last plan year	43	56	405	504
2.	External product balances	Physical units or forints	last plan year	10	15	50	75
3.1.	Wage-fund constraints	Forints	last plan year	3	-	-	3
3.2.	Live labour constraints	Persons	last plan year	4	-	15	19
4.	Land constraints	Cadastral acres	last plan year	-	7	3	10
5.	Capacity constraints	As constraint group 1.	last plan year	-	10	719	729
6.	Special technological proportionalities	As constraint group 1.	last plan year	-	-	45	45
7.1.-7.3.	Gross investment quotas: total; construction; domestic machinery	Forints	whole plan period	3	-	-	3
7.4.-7.5.	Gross investment quotas: imported machinery	Roubles, dollars	whole plan period	2	-	-	2
8.1.	Export and import constraints by products	As constraint group 1.	last plan year	-	-	661	661
8.2.	Constraint of machinery exports	Roubles, dollars	last plan year	-	2	-	2
9.	Foreign exchange balances	Roubles, dollars	last plan year	2	-	-	2
		All:		67	90	1898	2055

than users for their input. This accounts for the difference between the number of products and that of internal product balances.

2. External material balances. These limit the model's production variables in the input of external materials. The latter are products and services which are not the output of the model's production variables and whose total available quantity is given as constant. /In the case of some external materials, the import of the material in question from one or another market is treated as a variable and only the quantity available from other imports and from domestic production is given as constant/.

3. Live labour constraints. These limit the total input of wage-fund and labour. Separate constraints limit the input of male labour as well as the flow of labour between the agricultural and non-agricultural spheres.

4. The constraints of land. In the model there are six types of land available to the collective agricultural plants, viz. loose, semi-hard and hard ground, all three cultivated by dry or irrigation technology. In addition to these six agronomical constraints there is a seventh land constraint of socio-economic character: the household plot.

5. Capacity constraints. Production based on unchanging technology is limited by the available old capacities. The possibilities of reconstruction and plant enlargement are generally also limited. In some cases, due to special circumstances, the construction of new plants is also bounded from above.

In the four sectors of plant cultivation, special constraints were prescribed for the stock available in 1970 of the ten most important machine types.

6. Special technological proportionalities. These usually prescribe the technological ratios between the various production variables within some sphere of production. /E.g. the conditions of mixing in the chemical industry./

7. The constraints of investment resources. These limit the amounts available for gross investment - maintenance, reconstruction and the construction of new plant - over the whole plan period. The constraints are given in global form as well as separately for the input of construction, domestically produced machinery, socialist import machinery and capitalist import machinery.

8. Foreign trade constraints. All export variables are constrained with respect to every product, in order to express the foreign buyers' limited propensity to buy. Similarly, individual constraints by product were prescribed for every socialist import variable, in order to express the sellers' limited propensity to sell. On the other hand, no individual constraints were ordered to the capitalist import variables because here there is practically no upper bound to the propensity to sell, the constraint being represented by our own ability to buy. /The latter is expressed by the foreign-exchange balances./

An upper bound was prescribed also for the total export of machinery, to represent the difficulties of market expansion.

9. Foreign-exchange balances. Separate foreign-exchange balances were given for trade in socialist and capitalist markets. Export earnings constitute positive items in the balance. The negative items are made up of expenditure connected with the model's import variables /expenditure on priority, competitive imports/ and of import expenditure connected with the input by the model's production variables of non-priority products which do not figure among the 491

products of the model /expenditure on non-priority, non-competitive imports/.

The model has a total of 2055 constraints.

As in the case of defining the variables and the products, in the construction of the system of constraints it was also endeavoured to follow the index system of the National Planning Office and the ministries. This applies to constraint groups 1. and 2.[x/] which are closely related to the traditional system of product balances; to constraint group 3. which contains part of the traditional manpower balance system; to group 7. which follows the traditional breakdown of investment quotas. Due to the special characteristics of the model, the structure of constraint group 9. differs considerably from the traditional pattern of the balances of payment and of foreign trade; the differences can, however, be numerically explained.

Constraint groups 4., 5., 6. and 8. do not appear in direct form in the index system of traditional planning, though in actual practice the planners will endeavour to take into account the limits set by the area of land, the old capacities, and the possibilities of selling to and buying from foreign markets.

Let us now sum up what has been said above on the number of variables and constraints. The model describes the relationships of the five-year plan in a linear equation system which contains 2055 equations and 4479 variables /with the auxiliary variables included/. Linear programming has already been used in several countries for the purposes of economic planning, thus e.g. in France, India and Czechoslovakia. According to the available information, the Hungarian model is the largest

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x/ The term "traditional planning" is used here to describe the non-mathematical planning methods employed in the Hungarian planned economy for the last twenty years.

and most detailed of the economy-wide planning models known so far.

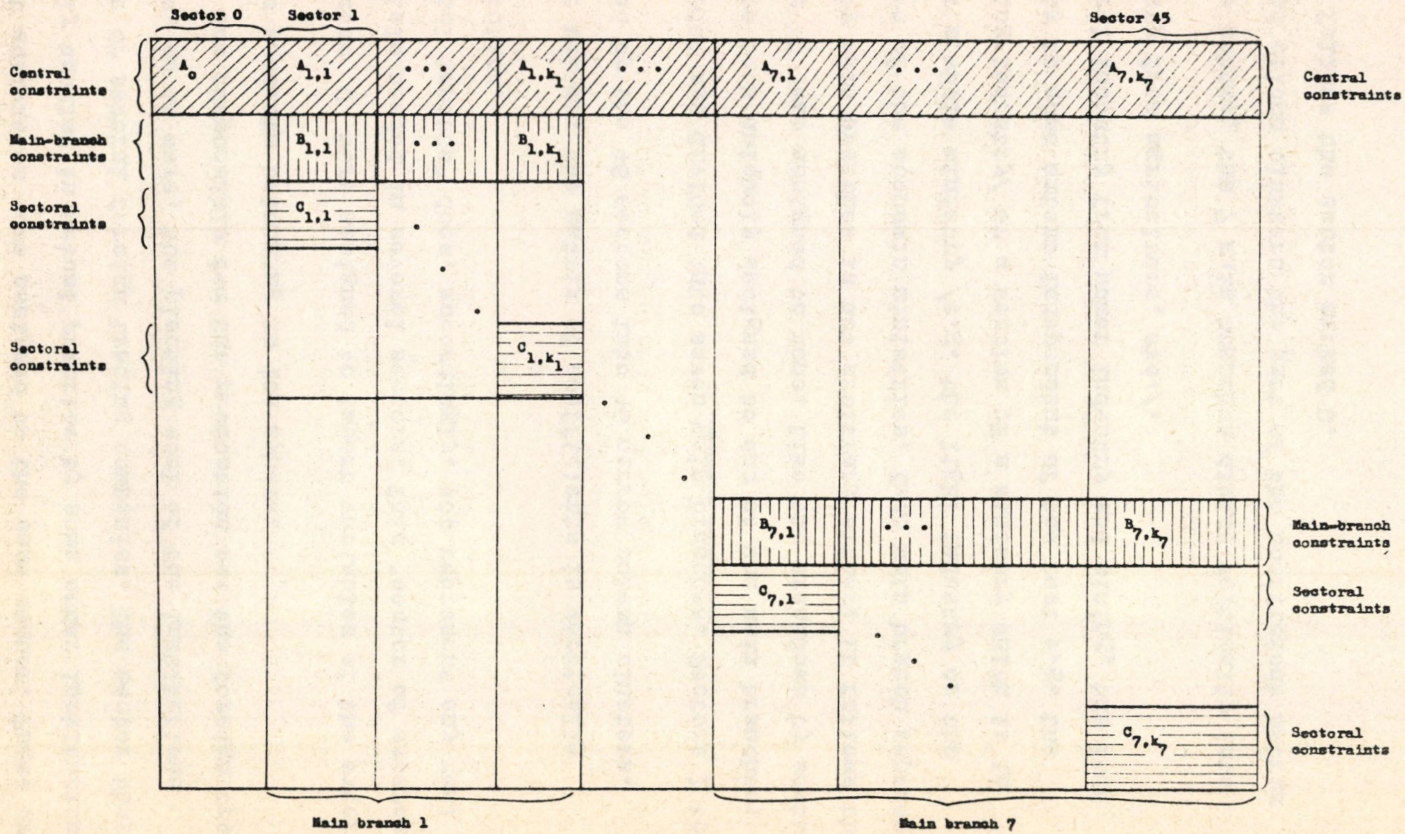
1.3. The "levels" of the system of constraints

In the foregoing, the constraints were classified according to their economic content. Let us now turn to another type of classification, that according to "levels". To facilitate understanding, the schema of the matrix of coefficients of the equation system is given on page 13 th.x/ This schema will be repeatedly referred to in the course of explaining the model's breakdown.

The model was divided into 46 sectors and every activity variable was assigned uniquely to one of the sectors. The breakdown is not arbitrary but reflects real economic contents. Sectors 1 to 46 represent each a sphere responsible for the production and foreign trade of a definite group of priority products. The productive activities of the sector generally represent one or several administrative units or institutions /industrial directorates, trusts, enterprises etc./. Thus e.g. the sector of the paper industry represents the Paper Industry Enterprise, the sector of the automobile and tractor industry represents the Motor and Tractor Trust, and so on. Together

x/ The schema contains only the coefficients of the activity variables; the unit vectors belonging to the auxiliary variables and containing the coefficients of the latter form a further block of the programming problem's matrix of coefficients, which is not represented here.

The three-level arrangement of the matrix of coefficients



with each productive activity, the corresponding export and import activities were assigned to the same sector. These are usually handled in actual practice by some other institution, by one or several foreign trading companies. The sector thus unites, as it were, the planning work of the institutions which are responsible for the production and the foreign trade of the products belonging to its sphere.

To sector 0 were assigned to import variables of the external materials needed in several sectors. This "sector of external material imports" does, accordingly, not represent any real institution.

In the schema, the matrix of coefficients is vertically divided by the 46 sectors into 46 narrow column clusters.

The model was divided into seven <u>main branches</u>. Sectors 1 to 45 were each uniquely assigned to one of the main branches; sector 0 was assigned to none. Like the breakdown by sectors, that by main branches is not arbitrary either; it follows the pattern of the economic ministries. Each main branch represents either a whole ministry /e.g. the light industry or the building industry/ or a section of a ministry which is in reality to some extent independent of the rest e.g. the chemical industry from power industry and mining, the food industry from agriculture, etc./.

In the schema, the 7 main branches divide vertically into 7 broad column clusters the part of the coefficient matrix which follows the sector marked 0.

After the vertical division let us now deal with the horizontal division. In the further description the sectors will be given a double suffix. The first suffix refers to the serial number of the main branch: $i=1,\ldots,7$. The second suffix gives the serial number of the sector within the main branch: $j = 1,\ldots,k_i$ $\sum_{i=1}^{7} k_i = 46$.

The contraints are grouped into three principal categories.

Sectoral constraints. These contain coefficients other than zero exclusively within the sector. To give a few examples:

- Capacity constraints of production in old plants with unchanged technology or reconstructed equipment.
- Export and import constraints of individual products.
- Internal balances of products which do not constitute an input for another sector of the model.

The sectoral constraints regulate the sector's "internal affairs" and its direct relations with the "world outside", that beyond the scope of the model.

In the schema, the blocks of the sectoral constraints are horizontally striped and marked $C_{1,1},\ldots,C_{1,k_1},C_{7,1},\ldots,C_{7,k_7}$.

The model contains a total of 1898 sectoral constraints.

Main-branch constraints. These contain coefficients other than zero in several sectors which belong, however, all to the same main branch. Some examples:

- The common land, labour and machinery constraints of agriculture.

- The common export constraints of the engineering industries.

- Internal balances of products which constitute the output of a sector and the input of another or several other sectors within a main branch but of no sector in the model outside the main branch concerned. /E.g. various chemical products within the main branch of the chemical industry./

The main-branch constraints regulate the main branch's "internal affairs", the mutual relations of its sectors, as well as its direct relations with the "world outside" that beyond the scope of the model.

In the schema, the blocks of the main-branch constraints are vertivally striped and marked $B_{1,1},\ldots,B_{1,k_1},\ldots,B_{7,k_7}$. The model contains a total of 90 main-branch constraints.

Central constraints. These contain coefficients other than zero in several main branches. To give a few examples:

- The constraints of live labour in the national economy as a whole.

- The investment quotas.

- Internal balances of products which are the output of a sector in one of the main branches but are used as input also in other main branches. /E.g. electric energy, wrapping paper etc./

The central constraints regulate the "common affairs" of the main branches.

In the schema, the blocks of the central constraints are diagonally striped and marked $A_0, A_{1,1}, \ldots, A_{1,k_1}, \ldots, A_{7,1}, \ldots, \ldots, A_{7,k_7}$.

The model contains a total of 67 central constraints.

In the terminology used here, the attributes "sectoral", "main-branch" and "central" indicate the "level" of the constraint.

<u>The present economy-wide model is, in its final form, a three-level one.</u>

In the table of constraints on p. 7., the number of constraints falling into the different levels is given for each constraint group based on classification according to economic contents.

<u>The multi-level structure is a particularly important characteristic of the present model. It is this property that differentiates it most clearly from the mathematical programming models used for economy-wide planning purposes in the past either in this country or abroad.</u>

1.4. The objective functions

In the series of computations surveyed here, several types of objective function were alternatively employed.

1. The maximization of additional consumption over and above "compulsory" private consumption. The composition of additional consumption was given with several product pattern variants.

2. The maximization of the surplus of the balance of capitalist foreign exchange.

3. The maximization of the surplus of the balance of socialist foreign exchange.

4. The minimization of total gross investment.

5. The minimization of live labour input /with two variants, viz. minimization of manpower and minimization of the wage fund/.

6. In some subsidiary calculations: the minimization of prime costs at current prices and of costs at computational prices.

In some computations /e.g. in parametric programming/ certain combinations of the above objectives were prescribed. Thus, the two types of foreign exchange balance were optimized jointly etc.

2. A system of models

2.1. Three model types - The union of models

Thanks to the special multi-level structure of the model described here, it is possible not only to employ it as a single large economy-wide model but also to carry out independent calculations by means of its blocks suitably separated from one another.

Three different model types have been worked out.

The sector model. This is used exclusively for programming the activities of a single sector - the j-th sector of the i-th main branch. The programming problem may be described as follows:

$$/1/ \qquad \begin{cases} A_{ij}\, x_{ij} = g_{ij.}^{(A)} \\ B_{ij}\, x_{ij} = g_{ij.}^{(B)} \\ C_{ij}\, x_{ij} = g_{ij.}^{(C)} \\ x_{ij} \geqq 0 \\ p'_{ij}\, x_{ij} \longrightarrow \max! \end{cases}$$

where

x_{ij} = the program vector

$\left[g_{ij.}^{(A)}, g_{ij.}^{(B)}, g_{ij.}^{(C)} \right]$ = the sector's constraint vector partitioned according to the central, main-branch and sectoral constraints

p_{ij} = the vector of the objective function coefficients

When constructing sector model /1/, the sector is separated from the whole of the national economy to which /and within which to the other sector models/ it remains connected by the appropriately determined vectors $g_{ij}^{(A)}$ and $g_{ij}^{(B)}$, the constraint constants belonging to the central and main-branch constraints.

As normally dimensioned, the sector models contain 30 to 80 constraints and 60 to 100 activity variables.

<u>The main-branch model.</u> This unites all k_i sectors of the i-th main branch. The programming problem may be defined as follows:

$$
/2/ \quad \begin{cases}
\sum_{j=1}^{k_i} A_{ij}\, x_{ij} = g_i^{(A)} \\
\sum_{j=1}^{k_i} B_{ij}\, x_{ij} = g_i^{(B)} \\
C_{ij}\, x_{ij} = g_{ij}^{(C)} & j = 1,\ldots,k_i \\
x_{ij} \geqq 0 & j = 1,\ldots,k_i \\
\sum_{j=1}^{k_i} p'_{ij}\, x_{ij} \longrightarrow \max!
\end{cases}
$$

where

$\left[g_i^{(A)},\ g_i^{(B)}\right]$ = the constraint vector of the main branch, partitioned according to the central and main-branch constraints.

When constructing main-branch model /2/, all sectors within the main branch are linked up with each other but the main branch itself is separated from the national economy as a whole. It remains connected with the rest of the economy /and within the latter with the other main-branch models/ exclusively by the appropriately defined vector $g_i^{(A)}$, the constraint constants belonging to the central constraints.

As normally dimensioned, the main-branch models contain 150 to 300 constraints and 300 to 500 activity variables.

<u>The economy-wide model.</u> This unites all sector models. The programming problem is the following:

$$
/3/ \quad \left\{
\begin{aligned}
& A_o x_o + \sum_{i=1}^{7} \sum_{j=1}^{k_i} A_{ij}\, x_{ij} = g^{(A)} \\
& \sum_{j=1}^{k_i} B_{ij}\, x_{ij} = g_i^{(B)} && i = 1,\dots,7 \\
& C_{ij}\, x_{ij} = g_{ij}^{(C)} && i = 1,\dots,7;\ j=1,\dots,k_i \\
& x_{ij} \geq 0 && i = 1,\dots,7;\ j=1,\dots,k_i \\
& \sum_{i=1}^{7} \sum_{j=1}^{k_i} p'_{ij}\, x_{ij} \longrightarrow \max!
\end{aligned}
\right.
$$

where

$g^{(A)}$ = the national economy's constraint vector ordered to the central constraints.

As mentioned above, economy-wide model /3/ contains 2055 constraints and 2424 activity variables.

The three model types differ from each other in the breadth of the sphere they cover - a single sector, a main branch composed of 3 to 11 sectors, or the whole of the economy with all its 46 sectors.

2.2. Decomposition - The connection of models

For the solving of the programming problem, two main methods offered themselves.

1. The direct solution. The problem may be solved directly, by means of an appropriate algorithm /e.g. the simplex method/. This method has been applied in every case to the sector models. However, in the case of the main-branch and economy-wide models this was not possible with the facilities available in Hungary.[x/]

2. The decomposition method. There are several decomposition methods known, thus first of all the Dantzig-Wolfe method[xx/] and those formulated on the basis of the theory of games.[xxx/]

Without attempting the full description or comparison of the various decomposition methods, let us survey some of their characteristic common features, especially as relating to the present special problem. For simplicity's sake, the exposition will be based on a two-level model instead of a three-level one.

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x/ With the present world level of computing techniques, this should not be impossible. Linear programming problems with several thousand counstraints have already been solved abroad with direct methods, without decomposition.

xx/ See Dantzig, G.B. - Wolfe, P.: "The decomposition algorithms for linear programs", Econometrica, 29 /1961/, 767-778.

xxx/ See Kornai, J. -- Lipták, Th.: "Two-level planning", Econometrica, 33 /1965/, 141-169.

In the direct solution, calculations are carried out simultaneously with the whole system of equations. /This means that in the present case certain simultaneous calculations would be carried out with 2055 equations./ With the application of the decomposition method, on the other hand, it becomes possible to reduce considerably the size of the equation systems handled at a time. These smaller equation systems may be classed into two main categories: higher-level and lower-level equation systems. The computations carried out with them may, accordingly, be termed higher-level and lower-level computations.

The four main criteria of the decomposition methods are the following:

a./ Instead of solving a single large equation system in a single calculation, several smaller equation systems must be solved several times.

b./ The higher-level computations are more aggregate in character while the lower-level computations are more disaggregate and more detailed.

c./ The method is iterative. In every iteration both higher-level and lower-level computations are carried out.

d./ In every iteration, the higher-level computations yield new information relevant to the lower-level ones and conversely. A flow of information in two directions is taking place, and it is these that provide a basis for repeated iteration on both levels.

The decomposition methods differ from each other in the realization of the above four criteria, namely

a./ in the principles of the model's decomposition and in the contents and size of the higher- and lower-level equation systems;

b./ in the degree and character of aggregation and disaggregation on the higher and the lower level;

c./ in the character of the computations to be carried out on the higher and the lower level;

d./ in the character of information flowing between the two levels.

Experience shows the known methods of decomposition to be extremely slow. Therefore, with the existing facilities of computing techniques in Hungary, they could not be employed in the project described here. Instead, an approximative decomposition method was worked out for the purpose. Its basic ideas were taken over from the Dantzig-Wolfe method; actually, it may be considered a "naive" version of the latter.[x/] It is but approximative in character; as opposed to e.g. the Dantzig-Wolfe method, it does not ensure the reaching of the exact optimum within a finite number of steps. On the other hand, it has the following advantages.

It sets to extensive use the heuristic logic of the planning economist and draws on the available additional information which is neglected by the exact method of calculation. Thus e.g. it turns to advantage the circumstance that at the beginning of the higher-level calculations one or even several sufficiently suitable /though not optimal/ solutions are usually already known. Thus, among others, the official program based on the traditional planning methods may be known. The approximative method leads in the first iteration already to a program which is certainly not less advantageous than the known solutions and most probably more advantageous

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x/ The method was worked out by the author. For a more detailed description see Information Bulletin No. 17 of the national economic programming project /"An approximative method of two-level planning", Budapest, 1966, Computing Centre of the Hungarian Academy of Sciences - Institute for Economic Planning, National Planning Office, mimeographed/.

than they are.[x/] In the subsequent iterations it will still further improve the objective function. As against this, the exact methods will yield an economically acceptable program after a great number of iterations only.

The approximative method was resorted to as a consequence of computing technical difficulties; it would therefore not be appropriate to make a virtue of this necessity. It is, however, a fact that in view of the uncertainty of the initial data no great importance is attached in planning to exact optimality in the mathematical sense of the term. The approximative method would therefore appear acceptable for practical purposes for the time being.

2.3. "System" - in what respect?

The use of the term system /in the system-theoretical, cybernetical sense/will be warranted if we are dealing with a structure or network of interconnected elements. In this sense, the methodology of the project surveyed here has resulted in evolving a model system whose elements are the various models.

The relationship between the elements can be approached from two aspects. One aspect was dealt with in paragraph 2.1., namely that the sub-models can be united with one another. With the combination of sub-models, "model building" can be formed. E.g. a "two-storey" building can be formed by uniting a group of sector models into a main-branch model or all 46 sector models directly into an economy-wide model. Or it is possible to form a "three-storey" building by uniting the 7 main-branch models /together with sector model 0/

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x/ Thus e.g. in the case of the economy-wide model, with the application of the approximative method an improvement amounting to some 150 million dollars could be achieved in the first iteration already in the objective function as against the official program.

into an economy-wide model.

The other aspect was discussed in paragraph 2.2. In the case of the application of decomposition methods, the sub-models are not united but connected, linked to each other by means of information flows between lower- and higher-level equation systems worked out in accordance with the concrete principles of the decomposition method.

Both the union of the sub-models and their linking up by means of information flows was made possible by the fact that all models are constructed on the basis of a uniform index system. Quantification was based in every case on strictly uniform nomenclatures, statistical definitions and classification.

It is an important property of the model system that there is unique communication between the higher-level and lower--level models and computations. The higher-level computations may yield, among others, aggregate economic indices, but it will in every case be possible to make detailed production, investment and foreign-trade programs uniquely correspond to them. And conversely, it will always be possible uniquely to derive from the detailed production, investment and foreign-trade programs aggregates established on the sectoral, main branch or economy level.

Here again, we have a characteristic feature of the methodology of the project which distinguishes it from the mathematical programming models used earlier for planning purposes. The projects known from Hungarian and foreign literature were all based on individual and independent models. It is for the first time now that an interdependent system of planning models, a network of aggregate and disaggregate models, a combined hierarchical structure of higher- and lower-level models could successfully be worked out.

3. The data of the model

The data and parameters employed in the calculations which serve to provide a foundation for economic decisions may be determined in various ways. Let us first describe below three pure cases and deal with their various combinations later on.

1. The phenomenon to be numerically described by the parameter is subjected to full-scale observation. This is the case e.g. when the inputs and outputs of a strictly defined economic unit /an enterprise, an economic branch, etc./ are determined on the basis of full-scale statistical observation.

In this case the economist carrying out the economic calculation will obtain the parameter directly from the <u>statistician</u> summarizing the data yielded by full-scale observation and will use it in his economic calculation without transformation.

Let us call this method the <u>simple economic-statistical method.</u>

2. There are some statistical observations available which do, however, not characterize directly the phenomenon to be described by the parameter of the economic calculation but admit only of indirect inferences. In drawing the indirect inferences, the tools of mathematical statistics are employed. For example, a trend calculated on the basis of a time series is extrapolated into the future. Or a confidence interval estimate is given, based on the mathematical statistical analysis of data distribution. Or again, an estimation of the parameter is worked out by determining an appropriate average value on the basis of representative sampling.

Here the economist carrying out the economic calculation obtains the data not directly from those carrying out and

summarizing the observations but from the mathematical statistician who processes their results.

Let us call this method the mathematical statistical method. /This is what in present-day Western terminology is called the econometric method./

3. There are some observations available, such as statistical data, technical or commercial information etc. which do, however, not characterize directly the phenomenon to be described by the parameter of the economic calculation but admit only of indirect inferences. The indirect inferences are drawn without using the tools of mathematical statistics, in a basically intuitive manner. Let us take, for example, an engineer who knows exactly the numerical characteristics of the technology employed at present and has also some information concerning the new technology to be employed in the future. He has a knowledge of the differences between the two technologies in qualitative, technical terms and may also have some numerical information obtained from the literature on the subject or in the course of personal consultations. Relying on these, he will make an estimate of the numerical characteristics of the new technology. Or take a foreign-trade expert who knows exactly the market situation, the price trends over the past years and the relevant sales data. He has an idea of the intentions of the buyers concerning the future, an idea formed in the course of personal contacts. Based on this information, he will give an estimate of future price formation and sales possibilities.

The estimates thus given may be primitive, inexpert, irresponsible; but they may as well be based on technical calculations and formulae, on the careful collection of information, worked out with expert knowledge and a sense of responsibility. In the latter case they will be more reliable, but still not "exact". Intuition will still play

a role and, as a consequence, the process of transforming the acquired information into an estimate will not lend itself to mathematical formulation, to description in exact terms.

In this case the economist carrying out the economic calculation obtains his data from the engineer, the foreign trade expert, the specialist.

Let us call this method that of expert estimation.

The differences between the three methods can, accordingly, be described as follows.

In the first case the data observed are built directly into the economic calculation, whereas in the second and third case they are made use of in an indirect manner. The transformation of the observations, takes place in the second case on the basis of mathematically formalized rules, by means of mathematical statistical methods, and in the third case without formalized rules, on the basis of expert knowledge.

In actual practice the three methods are frequently combined. For example, the data obtained by means of the first or second method may not be used directly in the economic calculation but corrected first by expert estimation to make them express more adequately the differences between the phenomena observed in the past and to be expected in the future. Or the third method may be basically applied but with the expert estimation making extensive use of full-scale statistical observations and information obtained by mathematical statistical tools /trend computations, various averages, functions quantified on the basis of econometric methods, etc./.

The majority of economic decisions is, in actual practice, reached through the third method, that of calculations based on expert estimation, frequently supplemented and combined with the first and the second method as described in the foregoing. This applies particularly to investment decisions, both under the conditions of the socialist planned economy and those of the capitalist market economy. As a matter of fact, in reaching such decisions extensive use is made of calculations relating to new, hitherto untested technologies as well as to price forecasts and future marketing possibilities.

Part of the mathematical models used in planning is based essentially on the first, simple economic-statistical method. Cases in point are the input-output tables quantified on the basis of full-scale statistical observation.

Another part of the planning models is based essentially on the second, mathematical statistical method. Characteristic cases are the econometric macro-models /the Dutch planning models, the Klein-Goldberger model worked out in the USA, the Hungarian M-1, M-2 models, and others./.

The quantification of the model described in this paper was carried out essentially by means of the third method, on the basis of expert estimation. This is an important characteristic which distinguishes it from a number of other planning models constructed in Hungary and abroad. The structure of the model - especially the great number of production, investment and foreign-trade variables - makes it imperative to rely to a great extent on the estimates of technical and foreign trade experts.

Only a comparatively minor proportion of the data was based on simple economic-statistical observation; that method was primarily used when determining the parameters of the variables of type 1. /the operation of old plants with unchanged technology/. In the case of a further data group, mathematical statistical methods were employed, thus e.g. in the calculations relating to trends in world-market prices, in working out the demand functions required to determine the pattern of consumption, etc. But even in these cases, the data obtained had to be corrected on the basis of expert estimates in order to make them suitable for practical application.

Some of the data used were taken over from the official documentary material of traditional planning work based on non-mathematical methods, in unchanged form or with corrections carried out in cooperation with experts. Other data, which could not be found in the official documentary material, were estimated by experts specifically for the purpose, and the specialists were in most cases those engaged in similar tasks in connection with the official planning projects. Accordingly, the quality of the estimates was presumably similar in the two cases.

The supply of information in traditional planning will be reverted to in the second paper. Here, it should suffice to state that the information basis of the model described in this paper was essentially of the same charecter as that of the traditional plans and of the investment calculations based on non-mathematical methods; as a matter of fact, the two methods partly share the same basis. The information material which would otherwise extensively influence the medium-range economic decisions but which is usually utilized in scattered form, without strict logical schematization, is united in the present model according to a uniform classification, in consistent and logical arrangement.

x x x

In conclusion, it may be stated that the first experimental computation series of economy-wide programming has proven the possibility of constructing a multi-level model system.

The fact is significant in itself. However, the proving of the possibility alone is not sufficient. The questions must also be answered, now can the method be employed in actual planning work; what is its practical purpose and role; what are the shortcomings of the first experiment; how should the model be further developed; what are the conditions of its systematic practical application? The answer to these questions should be provided in the second part of the paper.

PART II.

ON THE PRACTICAL APPLICATION OF THE MULTI-LEVEL PROGRAMMING MODEL OF THE NATIONAL ECONOMY

The main purpose of the experiment was to test a new method in mathematical programming.[x/] What possibilities of future application does the new method offer? In the course of the exposition it is not only the concrete realization of the first experiment that will be kept in view but also the task of further developing the mode. Similarly, the treatment of the model's "environment", of the traditional planning methods and of the relationship between mathematical and non-mathematical methods will not be confined to describing the present situation but the modifications that can be expected - or at least hoped for - will also be referred to. Utopian ideas, on the other hand, shall be avoided when speaking of the future. Only those changes will be dealt with which appear possible under the given objective conditions and whose realization depends fundamentally on implemented.

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x/ It was pointed out at the beginning of the research work already that the computations "...should be considered a scientific experiment the main importance of which lies in testing the new planning method... All this is emphasized here exclusively to make it clear from the beginning that the fact that the computations may provide a basis for practical planning decisions was considered of secondary and additional importance only ... It is deemed most important not to raise excessive expectations in leading economic circles concerning the immediate practical use of the calculations". Information Bulletin No. 1 of the national economic programming project, Budapest, 1963. Computing Centre of the Hungarian Academy of Sciences - Scientific Research Department, National Planning Office, mimeographed, pp. 64-65/.

4. The model's sphere of action

The multi-level economy-wide programming model embraces a broad sphere of the economy and is able to determine simultaneously several thousand plan indices. Its sphere of action may still be broadened by further development. It must, however, not be believed that either this or any other model could in itself perform all functions of planning. It is impossible to work out any "super-model" into which the ready data must simply be fed in order to retrieve a complete national economic plan.

4.1. Prognosis and preselection

The model described here belongs to the family of programming models. The members of the model family have the common characteristic of a clear distinction between the given conditions and the possibility of choice. When the structure and numerical material of a programming model is determined, it is at the same time determined what should be considered the given conditions /expressed by the constraints/ and what are within that the possibilities of choice /represented by the variables/. Instead of a single computation a whole calculation series can, of course, also be carried out, changing repeatedly either the constraints or the variables. It remains, however, valid for every single member of the series that at the beginning of the calculation it had already been determined what can be considered as variable and what cannot be considered so.

It follows that the planning functions of programming are complemented and preceded by two other functions: prognosis and preselection.

1. Prognosis gives an answer to questions relating to the future. What can be expected with certainty to materialize in the event, independently of the resolutions of the resolutions of the decision-makers? Where is it that the latter can interfere at all, and to what extent? What are the limits of interference? What are the expectable consequences of alternative economic activities? As can be seen, part of the prognoses is "unconditional" while part of them is dependent on certain "conditions", reckoning as they do with events and processes which would come about only if certain conditions were fulfilled.

Prognosis may be based on primitive forecasting; it may, however, be prepared also with more circumspection, with a careful analysis and mathematical-statistical examination of domestic time series, or on the basis of extensive international comparisons, or with special prognostic models.

2. In reality, an infinite variety of economic activities is possible. From among them, a finite number is chosen by preselection; these become represented in the model by the variables. /Thus e.g. some 2500 were chosen for the purpose of the first experimental model of the project described here./

Preselection may take place arbitrarily; important alternatives may be left out of consideration because of inadequate information or subjective bias. On the other hand, it may also take place on the basis an extensive collection of information and by means of special models worked out for the purpose.

The realiability of the programming model will depend to a high degree on the quality both of prognosis and preselection. Does the system of equations contain all the necessary and essential constraints? Have the equations describing the given conditions been determined numerically in the correct manner? Have not some already determined activities been considered as variables? Do the most significant and characteristic alternatives figure in the model? All this will depend on the efficiency of prognosis and preselection.

4.2. The model's "subject matter"

The model is used to work out recommendations and estimates relating to the economic activities of production, investment, technical development, product distribution in the productive sphere, and foreign trade. Let us call this the model's subject matter. Some further subjects, which do not fall within the subject matter of the model, should be mentioned.

1. The patterns of public and private consumption must be determined from outside for the purposes of the model. They may be given in several variants but their determination must take place outside of the model.

2. The wage-fund quota must be determined from outside. It will then be allocated to the branches by the model. This, however, is only part of the planning of income distribution, the rest must be planned outside of the model.

3. The labour quotas must be fixed from outside. These too will be allocated to the branches by the model. Again, this is only part of the work of manpower planning - the rest must be worked out outside of the model.

4. The outputs meant to serve the purposes of stockpiling are prescribed for the model. The suitable volume of stock must be established outside of the model.

5. The model has no regional aspects and does not provide recommendations for the territorial allocation of

the resources of production. Nor does it take into direct consideration the social consequences of economic development such as urbanization and the changes in social stratification. This will again call for computations to be carried out outside the model and for analysis not only in the economic but also in the sociological and other domains.

6. The model's sensitivity tests and shadow-price system provide important bases for price formation. The model itself is, however, no price model, leaving as it does out of consideration essential relationships which must in practical price formation be taken into account.

7. The model does not supply information about the suitable choice of the economic instruments, necessary to further the implementetion of the plan.

The subjects listed above will suffice to illustrate the fact that the subject matter of the present model does not embrace the entire subject matter of medium-term planning but only part of it.

The methods on which the planning of the subjects listed above as well as of those not mentioned here is based, is highly relevant from the point ov view of the quality of planning in general. It will make a great difference whether it is based on primitive forecasting or on more sophisticated methods such as mathematical statistical analysis, international comparison or special models of consumption, income distribution, labour planning, price formation, etc.

4.3. Parallel calculations and mutual control

Paragraphs 1.1. and 1.2. have dealt with the planning functions which complement those performed by the model described here. But besides complementarism there is also the need for some degree of parallelism, for calculations "competing" with one another.

The model - as all plan computations - is working with definite simplifying assumptions. It may prove useful to carry out parallelly also other calculations which are free from such simplification. The control calculations may even employ other simplifications. For example, the model is working with continuous variables and must therefore disregard the phenomena of indivisibility, above all others the fact that below a certain plant size it would be practically useless to set up a new plant. It will thus be worth while to carry out parallelly some plan calculations which do not disregard this circumstance and which are based either on intuitive methods of traditional, non-mathematical planning or on some procedure of discrete programming.

The data material of the model was based mainly on expert estimates, as described in Chapter 3 of the preceding paper. It is therefore necessary to carry out parallelly also calculations based on other sources of data, especially on full-scale statistical observation and on econometric estimates.

The parallel calculations are aimed at mutual control, at the reciprocal disclosure of errors. The results confirmed by both calculation series will provide a firmer basis for decision-making.

5. The requirements of rational planning and the model

In the preceding chapter sufficiently modest limits were set to the scope of action within which the model may perform its definite planning functions. The subsequent analysis will remain within the same limits.

In the course of the analysis, some requirements will be set to serve as guiding principles. In the view of the author, the work of planning may be termed mature and rational if it meets these requirements. Here, only the necessary conditions will be defined; these are, however, not sufficient to determine the complete requirement system of rational planning.

In the course of the discussion the requirement to be dealt with will in each case form an introduction to be followed by the explanations and the conceptual definitions.

Requirement 1. The plan should contain the basic regulation variables affecting the structure of the economy which are at the command of the "addressees".

The plan offers recommendations to certain institutions such as the central organs of economic administration, the ministries, the medium-level control agencies, large enterprises, banks, etc. Let us call these institutions the addressees of the plan. Every addressee has the power to regulate certain definite processes and magnitudes affecting the structure of the economy. What is meant here by regulation is that the trend of the process, the development of the magnitude in question depends basically on the addressee's own activities. The index which gives the planned and recommended value of the economic process or magnitude in question will be called a regulation variable.[x/] All other index numbers of the plan's index system, those which cannot be termed regulation variables in the terms of the above definition, will be called prognostic variables.

Regulation variables are, within the institutional framework of the Hungarian economy, for example the allocation of investment credits by branches, or investment proposals relating to major projects, or the export obligations that can be undertaken under a long-term foreign trade agreement. The growth rate of national income, on the other hand, is a prognostic and not a regulation variable. The government will not be able to determine the rate of growth; all it can do is to take certain steps which will in the long run affect its trend.

Under the conditions of the old /pre-1968/ mechanism,[xx/] traditional planning did more or less meet Requirement 1. The multi-level programming model of the national economy was - in constrast with a great number of other mathematical planning models - also constructed to meet this requirement. This is one of the main reasons of the model's highly detailed and disaggregate character. As a matter of fact,

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x/ In approximately the same sense, other authors would also use the terms "decision variable", "instrumental variable" and "action parameter".

xx/ What is called by Hungarian economists the "mechanism" is in fact the complex of methods of economic control. This comprises, among other things, the price system, the system of material incentives, the concrete forms of the planning system, the institutional framework of economic regulation. A comprehensive reform of the Hungarian economic mechanism is now under way; from the 1st of January, 1968 radical changes are to take place in the system of economic control.

it would otherwise not have been possible to build into the model the essential regulation variables of investment and foreign trade.

The opinion is widely held that it was only within the framework of the old mechanism that Requirement 1 had to be met. According to these views, under the new mechanism that is to take shape in 1968 the plan would have to contain only the "basic" and "principal" indicators. Any further breakdown of the plan - characteristic of the over-centralised old system of economic control - would become unnecessary.

These views are, in the author's opinion, erroneous. The reform of the economic mechanism will naturally necessitate the reexamination of the plan index system and thus also the revision of the regulation variables. Which institutions should be the plan's addressees under the new conditions? Which are the economic processes that the addressees will actually be able to control and regulate? By what means will the economic administration be able to achieve that the lower-level addressees of the plan also realize the planned values of the regulation variables addressed to them? These are questions which require careful analysis. But though the revision is justified, the requirement itself must be upheld. Any index system which does not meet Requirement 1 cannot be considered a plan and action programme, only a prognosis.

This means in practical terms that there is a continued need for planning models which contain in a comparatively detailed breakdown the major /partly centrally initiated/ investment proposals and the main items of long-range international trade agreements. In this connection too certain modifications may be necessary /e.g. the breaking down of investments by their financial sources/. This will, however, not affect essentially the degree of breakdown in the model and in the index system linked to it.

As a matter of fact, Requirement 1 represents but a reformulation of the traditional planning practice which gives "priority" treatment to the production of some particularly important commodity group, to the start of some particularly important group of investments etc. This practice must not be discontinued, for that would mean to relinquish the control over the structure of the national economy. To regulate every detail would clearly be impossible - nor should it be endeavoured. It is, however, both desirable and possible to keep the most important processes under control. "Priority treatment" should, of course, be based not on random choice but on selecting the processes which, together with their secondary effects, are to a great extent determinant of the structure of the national economy.

<u>Requirement 2. The plan should be comprehensive and should contain the principal aggregate indices of the economy.</u>

The traditional planning methods did more or less satisfy Requirement 2. So did also - and more consistently than traditional planning - the input-output tables used for planning purposes as well as the aggregate programming models based on them.x/ The multi-level programming model of the national economy, on the other hand, failed to meet this requirement in its first experimental application. In the course of further applications it should be possible to remedy these shortcomings. The model's index system will have to be extended to contain besides /and not instead of/ the partial "priority" production, investment and foreign-

x/ On the latter see e.g. Kornai, J. — Ujlaki L.: "The application of an aggregate programming model in five-year planning", Acta Economica, December 1967.

trade estimates, also the global figures of production, distribution, investment and appropriation. It is desirable that besides /and not instead of/ the physical indicators, the basic estimates of input and output, expressed in terms of value, also appear in the model.

This extension and amplification leads to a series of difficult methodological problems. A connection will have to be established between the physical indices of priority products and the figures relating to global values; the non-priority activities of a residual character will have to be computed and so forth. The solution of these methodological problems had already been under discussion for some time in planning circles, also independently of the development requirements of economy-wide programming.

Requirement 3. There should be communication between the aggregate and disaggregate plan indices.

The comparison and analysis of the consequences of alternative economic policies and the high-level decisions based on them can take place only on the basis of plan proposals conforming to Requirement 2, i.e. on that of aggregate indices. On the other hand, analysis and decision-taking will have to be followed, in accordance with Requirement 1, by the concrete definition of the regulation variables. It is this that necessitates communication between the aggregate and disaggregate indices.

With the traditional planning methods, Requirement 3 is hardly ever satisfied, though attempts are usually made at an approximation, at "breaking down" the economic policy characterized by the aggregate plan figures.

The majority of mathematical planning models would not undertake this task. It is one of the most important characteristics of the methodology of multi-level planning that - as has already been pointed out in the first part of the paper - in its own sphere of action it fully satisfies Requirement 3. To every higher-level, aggregate economy-wide program it will at any time be possible uniquely to order a detailed program - in the present case one of about 2400 variables - i.e. the disaggregate determination of the regulation variables of investment and foreign trade.

Requirement 4. The system of plan indices should be consistent.

In the course of the subsequent discussion a strict distinction will be made between the consistency of the system of plan indices, its freedom from logical contradictions, and the same property of the actual plan figures.[x/] The demands made on the former should be made clear by three sub-requirements.

Requirement 4/a. The system of plan indices should be logical.

The definitions, classifications, units of messurement, price factors, etc. relating to the system of plan indices should be defined so unequivocally and applied so logically that the operations /addition, multiplication, etc./ performed with the plan indices are strictly interpretable and the figures to be compared are really comparable, referring to the same sphere.

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x/ The system of plan indices may be conceived of as a set of blank forms, tabular schemas, together with the instructions for completing them. The plan figures represent the estimates to be entered in these forms. It is also possible to consider the system of plan indices an equation system written down merely in symbols, whereas the plan figures represent the numbers substituted for the symbols in the equations.

The requirement may seem trivial - it was none the less neglected up to now in practical planning. As regards the consistency of the index system, planning is decidedly lagging behind statistics and accounting.[x/]

The multi-level programming model of the national economy strictly enforces Requirement 4/A within its own sphere of action.

Requirement 4/B. The relationship between the various plan indices should be explicable and deducible.

When formulating this requirement, the question as to the degree of exactitude in the deductions and explications was left open. All that is required here is the ability to describe the logical process leading from one plan figure to the other, or, in general terms, that leading from some information input through the plan computation to the information output.

Traditional planning is able to deduce and to explain many types of relationship, at least verbally or in the form of simple equations and balances. A great number of other relationships will, however, remain unexplained. They will

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x/ Some years ago it was suggested at the National Planning Office that the handling of figures should be put in order and that, as a means to this end, the supply of planning data should be mechanized. For the purpose, standard forms were prepared, based on a logical code system. The idea was not to provide some "scientific basis" for the plan but to introduce a data handling system the existence of which would be considered an elementary necessity in any properly managed modern plant. Unfortunately, the scheme was realized in a single partial domain only, but the experiment has proven the feasibility of the arrangement of data intc a uniform system and of mechanical data processing. However, apart from this, the scheme of the unification and mechanical processing of the supply of planning data proved a failure.

possibly not be subjected to deduction, not even mentally, or, if so, not described, and their explication not sufficiently controlled. The national economic plan is usually not supplemented by a detailed explication and documentation of the plan computations.

Economy-wide programming carries out, within its own sphere of action, a strict deduction of the relationship between the plan indices. It can be reproduced at any time how the information output /the program/ had arisen from the information input /the coefficients, the constraints and the objective function/. The application of economy-wide programming may thus mark a significant progress towards satisfying Requirement 4/B.

> Requirement 4/C. The plan computations should describe as completely as possible the relationships and proportionalities adherence to which is essential for the implementation of the plan.

Requirement 4/B calls for the simple explanation of the plan figures. Requirement 4/C goes farther than that, calling as it does for the most extensive and complete description of the relationships.

Mathematical programming is strictly taking into account all relationships that had been built into the model, neglecting at the same time completely the relationships which do not figure in it.

With traditional planning the case is entirely different. The National Planning Office, the ministries and the various enterprises have several thousand experienced planners on their staff. Taken together, these planners have actually considerably more relationships in view than the largest mathematical model - not a few thousand but tens or even hundreds of thousand. On the other hand - as pointed out above in connection with Requirement 4/B - these relationships would in their majority not be documented. Accordingly, it will usually not be possible to ascertain whether the relevant relationships had been taken into account or not.

The tasks to be carried out to satisfy Requirements 4/B and 4/C are closely interlinked. The problems lie in several directions. There would be a need to widen the sphere of documentation on plan indices, plan relations and planning computations, also apart from the needs of the mathematical planning projects. The relationships which had in many cases been living up to now in the thoughts of individual planners only, never finding verbal expression, or were, at best, stored in desk drawers among the mass of computational material, ought to be put in writing and made accessible to the other planners as well.x/

At the same time the system of constraints in mathematical planning should also be made more complete. Besides Requirement 1 /the establishing of regulation variables/, Requirement 4/C, <u>the control of the plan's workability,</u>

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x/ To avoid all misunderstanding: what is called for here is only the widening and ordering of the <u>documentation</u>. This should be strictly separated from the following questions, not to be dealt with in the present paper;

- From the wide range of plan targets, which are those requiring legislative sanction?
- From the range of plan targets and computations, what should be made public?
- What are the plan targets whose implementation must be promoted by central measures?

also warrants great detail and a comparatively high degree of disaggregation in the mathematical planning model. The aggregate models may more easily shift towards unworkable sectoral patterns than the multi-level model which takes into account the conditions of foreign trade, technology, natural endowments etc.

The constraint system of a mathematical model can, of course, never be entirely "complete". The limitations are partly of computation technical character. The programs yielded by the mathematical methods should therefore always be checked upon by practical experts. It should be tested whether the program which is feasible from the model's point of view is also consistent with certain proportionalities, conditions and relationships which are left out of account by the model but sensed by the practical planner.

Requirement 5. The plan should be workable and ensure an equilibrium.

Requirement 4 has called for consistency and freedom from logical contradictions in the system of plan indices. The present requirement calls for freedom from contradictions in the plan targets as well.

The concept of equilibrium is to be interpreted here as follows.

There should be no definiency of the products and resources taken into account in the system of plan indices. There may be a surplus, but the planners should know its extent and take into consideration its consequences /such as the accumulation of stocks, the unemployment of certain groups etc./.

As a consequence of unforeseen circumstances, the actual extent of the resources or inputs may in reality differ from that envisaged in the plan. Nor does Requirement 5 raise the unrealistic demand of a complete realization of the plan. What it does, however, call for is that in the course of planning, in possession of all given and available information, every factor is carefully taken into account which may impede the plan's implementation. A relative equilibrium of all relationships in the plan should be attained as far as the supply of information permits should be attained by means of minimizing the potential equilibrium disturbances.

The multi-level programming model of the national economy, with its highly disaggregate structure embracing some 2000 relationships, strictly enforces Requirement 5 within its own sphere of action. At the same time, in the first concrete model of the experimental computatuons several equilibrium and proportionality relationships were neglected the taking into account of which would have required summarization in value terms. As pointed out already, with the new models to be worked out later on it should be possible to eliminate this definiviency.

As far as Requirement 5 is concerned, economy-wide programming has a marked advantage over the traditional planning methods. No intuitive method of plan coordination, no repeated verbal or written discussion can compete with the strict internal logic of a mathematical equation system.

Requirement 6. The plan should be at least approximately efficient.

A plan will be called efficient if it is workable, ensures an equilibrium /i.e. satisties Requirement 5/, and cannot be confronted with another equally workable plan which is not less advantageous from any point of view and more advantageous from at least one. For example, both Plan 1 and Plan 2 may be efficient if one envisages higher private consumption and a less favourable balance of payments and the other lower private consumption and a more favourable balance of pyments. On the other hand, Plan 3 will be non-efficient if it is less advantageous than say Plan 1 from the point of view of private consumption and the balance of payments. In that case, Plan 1 will <u>dominate</u> Plan 3.

Traditional planning can not even approximately satisfy Requirement 6, if for no other reason than that it will not be able to fulfil either Requirement 4 /particularly not 4/B/ or Requirement 5. There can, however, be no question of efficiency if a plan's estimates are not explained and deduced in every detail, its index system is not consistent, and its equilibrium can therefore not be demonstrated. Nor can it be ascertained under those circumstances whether it does or does not dominate the other plans that it may be confronted with.

Multi-level economy-wide programming will be able to satisfy Requirement 6. True, it will only approximate full efficiency. First, as has been pointed out when discussing Requirement 5, it does not ensure complete workability. Second, it has to employ an approximate method instead of an exact procedure. It has nevertheless succeeded in working out plan proposals which are from several aspects significantly more advantageous /representing as they do conisderable savings/ than the plans based on the traditional methods.

The table below presents the results of the four programs worked out on the basis of the economy-wide computation series. None of the four programs dominates any of the others and all are approximately efficient.

Savings

	Saving as agains the permitted quotas		Over-fulfilment as against prescribed balances of foreign exchange	
	Investment /billion forints/	Labour /thousands of man-power/	Socialist /million roubles/	Capitalist /million dollars/
Program minimizing investment inputs	15.6			
Program-minimizing live-labour inputs		25.3		
Program optimizing the balance of socialist foreign exchange			126.8	
Program optimizing the balance of capitalist foreign exchange				125.9

With the further development of the economy-wide programming model, the improvement of computing techniques and the application of exact procedures it should be possible to make a progress towards the replacement of the present approximate computations by methods satisfying Requirement 6.

Requirement 7. The planning organs should submit to the political decision-making bodies the political plan variants made ready for decision. The variants should afford a possibility of selection in conformity with the timely political decisions and present the consequences of the alternative choices.

What is called here the political plan variants are the plan variables which differ from each other in their essential political consequences, as regards e.g. the planned standard of living, the rate of increase in the production fund, the orientation of foreign trade and of international credit policies etc.

A plan variant may be termed ready for decision if it satisfies the six requirements, dealt with above. This will make it possible for the supreme decision-making bodies to survey the basic variants every one of which is in itself consistent, workable and nearly efficient and can be "translated" into regulation variables, i.e. into indices whose regulation is actually in the hands of the State.

Traditional planning is not able to prepare simultaneous political plan variants. Multi-level economy-wide programming, on the other hand, is able to do so within its own sphere of action. It is here that lies one of its main purposes. A great number of economy-wide plan variants has already been prepared in the course of higher-level computations. The methodology employed in these computations makes it possible for every variant to satisfy in itself the constraint system described in the model and to be approximately efficient. Moreover, it will be possible to give for every variant characterized by its principal indicators a detailed program broken down by regulation variables.

Requirement 8. The period of regulation should be as short, the time horizon as long as possible.

To formulate this requirement two new concepts have been introduced. The first is the period of regulation. By this is meant the period of time for which the planned value of the regulation variables basically affecting the structure of the economy is fixed in advance. In the definition two words are stressed. "Fixed" - i.e. the planned value of the regulation variables will not be changed within the regulation period. "Planned" - i.e. fixing refers only to the suggested value of the regulation variable, and the possibility is not excluded that in the event the actual magnitude will differ from the planned one.

The second important concept is that of the time horizon. This is the period of time for which the consequences of the planned value of the regulation variables are estimated and forecast.

The logical point of departure of Requirement 8 is that the value of the structural regulation variables should be determined in the most reliable manner. This requires, on the one hand, that they are based on the most recent information, e.g. that the investment decision is as close in time to the starting of the investment project as possible /short period of regulation/, and, on the other hand, that the lasting consequences and their interrelations are considered in the most careful manner /long time horizon/.

In the ideal case the period of regulation will be one year and the time horizon between 15 and 20 years or, in certain relations, infinite. For the time being, as a first step in the development of planning methodology, less favourable solutions may also be termed satisfactory. For example, the regulation period may be between 2 and 3 years - the working out of a five-year plan, with a revision of the plan around the middle of the period. The time horizon may be 12 to 15 years - 15 years at the time when the original five-year plan is being worked out, and the remaining 12 years at the time of its revision.

Traditional, non-mathematical planning has made a start in this direction. Around the middle of the term of the present five-year plan, its revision was begun. Preparations were started for the so-called long-term plan which could give an adequate time horizon to the new five-year plan covering the years 1971-75.

The multi-level programming model of the national economy is even in its first formulation suited to satisfy the first half of Requirement 8. /This question will be reverted to when discussing Requirement 9 below./ The first experimental computation, however, didas yet not have a lengthy time horizon reaching beyond the regulation period. The model will have to be developed in a way that it has a time horizon reaching beyond the five-year term - say one of 15 years. It should not be necessary to give the model for the second and third period the same detailed breakdown /some 2400 variables/ as for the first one, as it is only for the first period of regulation that the value of the regulation variables has to be computed.

The methodological problems of the model's dynamization, of its transformation into a multi-periodic model, cannot be dealt with here.

<u>Requirement 9. Planning need to be continuous. The methodology of planning should enable the continuos processing of fresh information and a speedy revision of the plan computations.</u>

From Requirements 7 and 8 there follows logically Requirement 9, that of <u>continuous</u> planning. Reality is continuously changing around ourselves; there is a continuous change in the available technical, economic and political information, in the procedures and forecasts relating to the future, in the instructions received from the political bodies, and so forth.

There is need for a planning methodology which considers the continuous modifications in the information material the natural way of things. The structure of the system of plan indices should be comparatively stable. Its classifications and nomenclatures should be as permanent as possible; the definitions of the indices should possibly not change but remain comparable; nor should there be major changes in the system of data supply and in the arrangement of the forms, etc. The numerical contents of the system of plan indices should also be as up-to-date as possible. The continuous revision of the already completed plan computations should be ensured. Information processing and the computation of secondary effects should be quick; the procedures of plan coordination, revision and variant computation easily and speedily repeatable.

Traditional planning is practically unable to cope with Requirement 9, although this would be needed over and over again. Complaints like "Everything has again been upset..." or "The data supplied by this or that institution, person, agency etc. is again different from that received last time..." are frequently heard. The elaboration of a five-year plan generally takes several years, and although simultaneous variants are not worked out, the plan will in the course of time and planning repeatedly suffer modifications. Yet it will normally not be possible to carry out a consistent correction of the earlier plan proposals, to work out systematically all secondary consequences of partial changes. This task is practically insoluble by means of "handicraft" methods.

The methodology of economy-wide programming, on the other hand, affords a means to perform continuous planning. A model is being worked out, together with the index system belonging to it. All data pertaining to the model are stored on punched cards, punched tapes and magnetic tapes. The individual partial computations can be performed with extreme speed. In the case of a change in any data or data group, it will be simply changed at the corresponding place of storage /the old punched card will be replaced by a new one etc./ and it becomes immediately possible to assess the consequences of the modification.

It is exactly here that lies the main strength of mathematical planning - in the ability to recompute a plan in the possession of the new information material. This, however is but a methodological possibility, the utilization of which will depend on personal, organizational and computing technical conditions. These will be dealt with in greater detail later on.

6. The practical conditions of systematic application

These paper is not intended to give rise to false illusions. Multi-level economy-wide programming is not pretended to provide a solution for every planning problem. It may, however, be taken for certain that - and this has been convincingly proven by the first experimental computations - the application of the model described here would significantly advance the development of Hungarian economic planning.

The first computation took considerably more time than originally expected. It lasted some four years, not counting the periods of theoretical preparation and of the ulterior evaluation in detail of the completed computations.x/ Practical application of the method will be possible only if the next computation can be carried out within a much shorter period of time. The results concerning the economy as a whole will have to be available already before the beginning of the regulation period, at the time when the plan decisions are actually made.

Let us now survey the personal, organizational and computing technical problems of practical utilization and of speeding up the computations.

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x/ Partial results such as the recommendations of the sector-level computations could be drawn upon while the work was still in course and utilized for the purposes of the five-year plan. The economy-wide results were unfortunately obtained rather late. They will therefore be used only in indirect form, for the revision of the plan and, in general, for working out the economic policies of the future.

6.1. The machinery

At the height of the first experimental computation the project engaged 6 to 10 full-time research economists, 4 to 5 full-time computing-technical mathematicians, 12 to 15 part-time economists, 4 to 5 laboratory assistants, joined by some 150 to 200 occasional participants, such as data suppliers, advisers and those carrying out partial tasks. This large panel was composed of members belonging to about thirty different institutions - scientific institutes, computing centres, high- and medium-level economic institutions and enterprises.

The collective was composed exclusively of voluntary participants. Every one of them was invited personally and asked first whether he or she would be inclined to join in. Only when this was cleared would the invitation be "legalized" by asking the permission of the superiors of the person concerned, the official authorization of the institution in question to participate.

With this team recruited of volunteers a high degree of centralization could be achieved. Methodological unity was ensured more strictly than in the case of the traditional index system which, although enforced by State discipline, is less logically consistent. And all this in spite of the fact that those directing the research project had no "authority" whatever to demand the enforcement of the methodology.

The driving force behind the project was a strange mixture of team-forming motives. It contained the elements of a "movement", the optimist's belief in a fine idea, that of rational mathematical planning, and expressed in enthusiasm, unselfishness and voluntary discipline. It contained also the elements of the scientific "team", the joy of joint intellectual excitement, of joint discovery and thinking, as a driving force. And there is no reason to deny it; the elements of "side work", so characteristic of present-day conditions, were not missing either, as a considerable part of those participating made some side money from the project.

From what has been said it will be clear that it should not be possible to repeat the project in this form. Enthusiasm will not last forever; towards the end of the work even that of the most unselfish participants showed signs of abating. Scientific interest is also bound to decline when the excitement of discovery is over and the problems are confined to the practical application of a novel method. Finally, as regards the "side work" aspect, this type of work is certainly not suited to provide a lasting basis for the systematic application of a specially important method.

Economy-wide programming will have to abandon the working methods characteristic of a "movement", a "scientific team" and a "side activity", and adopt those of official work - preserving, of course, to the highest possible degree the enthusiasm and intellectual standard of the former. Economy-wide planning should be changed from a "non-professional" to a "professional" activity.

Voluntary discipline is highly preferable to enforced discipline, as long as there is a will to work in a disciplined way. However, should this will be lacking, there will be no possibility to interfere. In the course of the project, delays ere frequently caused by some participants' slack-ness and lack of discipline, or by the fact that they were assigned some other task by their own institution or superiors. It is a characteristic fact that the first sector computations were finished in 1964 already - which proves that it is possible to complete a sector model within three quarters of a year. Yet there were some sectors whose model was not ready till two and a half years later, early in 1967.

Should the economic administration intend to have computations of this character completed in the future within a considerably shorter period of time, then it will be necessary to bring into existence a machinery of suitable dimensions whose main purpose would consist in carrying out economy-wide programming and which could be compelled to observe the time limits set for its tasks.x/ This machinery should be served by experts well versed in mathematical planning methods and operate within the central and medium-level planning organisations themselves.

6.2. The standard of knowledge of the practical planners

At the start of the research project, the training of programming experts had been set as one of the primary objectives. The results achieved may be termed satisfactory. In the beginning there were probably three or four participants who had previously already been engaged in practical mathematical programming, at least on the sector level. The rest had some knowledge of the method from literature, or none at all. By the time of the project's conclusion, at least 40 to 60 research workers were thoroughly trained in the methodology of economy-wide programming, not only theoretically but on the basis of practical experience as well.

In the knowledge of mathematical planning - as in the learning of languages - there are two degrees to be distinguished: active and passive knowledge. The above-mentioned 40 to 60 participants in the programming project

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x/ It would have been more fortunate if this machinery had come into being at the time of the first experimental computations already. Then, the torch of the task could have been passed on, as it were, by the researchers to the practical planners. They could have been going together at least half of the path, and the future practical users could draw also their own experiences. No ulterior transfer of working methods, no teaching by word or writing will ever make up for experiences of this type.

have attained active knowledge and should be able in the future to construct a model and to carry out computations with it. It is, however, necessary that the greatest possible number of practical planners acquire at least a passive knowledge of the method. As has been pointed out above in detail, the model is closely linked with its environment, with the work of planning as a whole. It is the practical planners who supply the data and who have to assist in the construction of the model - in the selection of the variables as well as in working out the system of constraints - and in the practical evaluation of the results obtained. It is their task to put the questions to the model and to process the answers received. All this requires at least an elementary knowledge of the model's language. In other words: every practical planner should know the conceptual system and general logics of mathematical planning /even if he has not mastered the technique in all its details/.

Unfortunately, in this field but little has been achieved. Experience has shown that the ideas of mathematical planning have become but little absorbed by the many thousand central, medium-level and enterprise planners. The material has not been extensively studied, nor the literature on the subject. Analysis of the reasons would lead too far; suffice it here to state the fact.

A radical change in the training and retraining of practical planners is an essential precondition of the systematic practical utilization of multi-level economy-wide programming and of mathematical planning in general. The up-to-date planning methods should be taught extensively and systematically.

6.3. The supply of data

The economy-wide programming model has been constructed to conform as far as possible to the index system of traditional planning. This was done to be able to utilize to the greatest possible extent the information material of the Planning Office and the ministries.

In retrospect, it may be stated that this procedure was correct. It would have been almost impossible to procure all the model's data unaided. And even so, the task proved more difficult than expected. Considerably more data than expected had to be collected from sources outside the official documentation or worked out specially for the purpose. This was one of the main reasons of the work's protraction.

What exactly were the causes of the difficulties in data supply and how can they be eliminated in the future?

One problem has already been pointed out in connection with consistency, Requirement 4 of rational planning. The whole of traditional indices does not form a coherent system free of logical contradictions.

The systematic application and further development of economy-wide programming is inseparably connected with the development of the plan index system as a whole. Mathematical modelling which, with its requirement of consistent data supply, is pressing for a general regulation, may as well become one of the beneficiaries of the result.

Even independently of the problems of multi-level mathematical programming, the general development of the plan index system is now the order of the day. The system of indices employed up to the present needs thorough revision. Stable definitions and classifications should be worked out, and uniform forms and data documentation prescribed. And all this should be carried out in a way to coordinate completely the data requirements of statistics, non-mathematical planning and the essential plan models. This would enable at the same time the mechanical data processing of the plan information material.

A further problem is posed by the fact that in traditional planning but few data are collected which could be used for the computation of variants, be it lower-level variants /e.g. investment projects based on alternative technologies/ or higher-level variants /alternative economy-wide plans/.

In connection with the data, mention should be made also of the problem of information responsibility. A recurring experience was the following.

A figure, let us say an export constraint, was supplied by a member of the official planning staff. It was inserted in the model, the computation was run through, and it turned out that the program exhausted the constraint and the export figure reached the permitted maximum. Thereupon, the planner who had supplied the figure quickly withdrew it, saying that it was in fact impossible to reach. It is one of the most important characteristics of mathematical planning that the model, the computer, cannot differentiate between the seriously founded figures and those irresponsively thrown in - it will deal with every individual figure in the identical manner. Those in charge of official planning work have in the course of time developed a certain instinct to deal with the figures submitted: to increase or decrease them according to what they are referring to and by whom they were submitted. The computer has no such instinct. This was a situation that proved rather unusual to those supplying the model's data and, as a consequence, a great number of computations had to be repeated.

When all is said and done, economy-wide programming will require neither more nor other data than traditional planning would within the same sphere of action. After appropriate coordination it should be achieved that all information required for the quantification of the multi-level programming model are collected through the channels of official plan data supply.

6.4. Computing techniques

Probably the main reasons for the protraction of the first experimental computations lay in severe computing technical difficulties. Nor were these difficulties of the same character as those usually encountered by pioneers in some field, such as the first steamboatmen or flyers. As a matter of fact, it should have been possible to provide for the necessary computing technical conditions on present-day standards, from the country's given resources /and even from the foreign exchange spent on computers all over the country in the course of the past few years/ and with the existing Hungarian staff of mathematicians and computing technicians.

In the course of the project's four to five years, six different computing centres had to be used. As it was - correctly, too - endeavoured to employ at every given time the largest computer available in the country, the type had to be changed three times. Each time the laborious procedure of working out and running in the computer programs had to be started again. All this was still aggravated by the fact that economic planning in Hungary has no computing technical base of its own; there is no computer in the country to serve primarily the purposes of planning. The economy-wide programming project had to be carried out on computers belonging to various institutions which were sometimes cooperative and sometimes less so. This has slowed down enormously the working out of new computer programs and the computations based on them.

It is an essential precondition of the operative application of economy-wide programming /and of mathematical planning in general/ that the planning apparatus should be served by an adequate computing technical base of its own: a large-size computer technically suited for the quick and reliable solution of extensive programming problems with high data requirements, together with an appropriate staff of mathematicians and attendants. As a matter of fact, these are quite trivial conditions the realization of which should not encounter any objective difficulty.

6.5. As a function of the general development of planning

In sections 3.1.-3.4., a survey was given of the factors which lengthened the execution of the first experimental computation as well as of the conditions which are essential for the method's further /and speedier/ application. Now, will these conditions be realized?

There are certain economic models which may be worked out in the quiet rooms of a research institute, quantified on the basis of printed statistics, and published together with their results in periodicals without the research workers' ever having to come into contact with the men of practice. The multi-level planning model is not one of these. It is linked with a thousand threads to the living machinery of planning, from the data requirements and the practical advice needed for its concrete construction to the utilization of the results. The model will either succeed in fitting organically into the living work of planning or will become eliminated in the course of time.

Its destinies were up to now mainly in the hands of research workers who had initiated the work and carried out the first experimental computations. Up to now it needed only benevolent backing on the part of practice - and this was usually granted.[x] Now, however, a new phase has been reached: that of application. The destinies of multi-level economy-wide programming have passed into the hands of the practical planners, first of all into the hands of those responsible for the control of planning work. The mathematical economists and the research workers will continue to play an important part; they will have to help in training the staff and in developing the methodology. But it will obviously lie beyond their power to organize the official machinery required for the practical application and the large-scale retraining of practical planners, to transform fundamentally the system of plan indices and to build out the computing technical base of planning. All this should be left to economic administration, to those in control of the planning machinery.

The paper was dealing throughout with the conditions of the application of multi-level economy-wide programming. The problems are, however, closely interrelated with those of the general development of planning. It is not only the present model that requires the presence of mathematical planning experts in the official planning machinery, the up-to-date retraining of practical planners, the reorganization of the supply of planning data and the setting up of a computing technical base for planning. All this has by now become timely anyway. The practical application of multi-level economy-wide programming will be but a <u>function</u> /and, at the same time, a clear <u>measure</u>/ of the further progress towards rendering planning more rational and raising its intellectual standard.

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x/ The acknowledgements of the collective of economy-wide programming are due primarily to the National Planning Office for the financial support given to the project; to the Computing Centre of the Hungarian Academy of Sciences for the work devoted to the solution of the computing technical problems; and to all the institutions - ministries, research institutes, computing centres and enterprises - which have helped their activities in various forms.