MULTI-LEVEL PROGRAMMING —A FIRST REPORT ON THE MODEL AND ON THE EXPERIMENTAL COMPUTATIONS

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PREFACE

The present study is a translation of a paper published in *Közgazdasági Szemle*, 1968, Nos. 1 and 2. Part I was entitled "A Multi-Level Programming Model of the National Economy" [A többszintü népgazdasági programozás modellje], and Part II – "Practical Application of the Multi-Level Programming Model of the National Economy" [A többszintü népgazdasági programozás gyakorlati alkalmazásáról].

* * *

We have completed the first experimental computing sequence for the multi-level programming of the national economy. We should like to publish the various experiences of our work in the fields of economics, methodology of planning, and economic policy in detailed studies and later, possibly, in the form of a book. However, evaluation of results and preparation of publications require much time. We therefore publish this report to outline the structure of our model and to discuss problems of the future implementation of our method.

PART I

A MULTI-LEVEL PROGRAMMING MODEL OF THE NATIONAL ECONOMY

The idea of linking the various sector models and uniting them in a single economy-wide programming calculation was first put forward in Hungary seven years ago. Two years later, the practical preparations for a national programming project were launched.[1] Since that time, a large team of theoretical economists, practical planners, mathematicians, computing technicians, and engineers has been active in the project; at the height of the work they numbered several hundred.[2] The aim was to put to practical test, on the basis of factual figures, the method of multi-level planning.

This aim was accomplished, although the realization took more time and encountered greater difficulties than had been expected. *The five-year economic plan's multi-level planning model, suited to mathematical programming, has been completed and tested in practice.* True, it is only a "prototype" that should be further improved to meet future requirements.

Despite this, it has come to life, not only in theoretical papers but with figures, on punched tapes, cards, and magnetic tapes that can be fed into the electronic computer. Moreover, the first computation series based on the central model resulting from the union of the sector models has already been successfully carried out. As regards the practical consequences of the economic policy to be drawn from the computations, a report has been presented and extensively discussed at the National Planning Office.

Some complementary calculations remain to be worked out. It may take several years to analyze the lessons to be drawn from long years of research work – lessons that will benefit both economic policy and the theory and methodology of planning. This paper will give a short description of the model's structure and information basis; the problems of application will be dealt with in another paper. A more detailed and complete analysis will come later.

1. STRUCTURE OF THE MODEL

The method employed represents an application of *mathematical programming* in economic planning. In the first experimental calculation the simplest form – linear programming – was used. In the future, as soon as computing-technical facilities permit, we can change to other programming methods containing discontinuous variables and nonlinear relationships that are more adequate from the point of view of economic theory.

Section 1.3 of this paper deals with the model's special "multilevel" structure. In the first approach of the description this aspect is disregarded, and the economy-wide model is treated as a single large linear programming problem.

1.1 THE VARIABLES

The model performs the programming for the production, distribution, and foreign trade of 491 *products*. These are generally not concrete and fully specified commodities, but rather product groups or aggregates, such as "coal and anthracite," "block aluminium," "boring machines," "bricks," "cotton-type yarns," "canned meat," etc. In defining the products, we followed the nomenclature of "priority products" used in the planning work of the National Planning Office and the ministries.

The production and foreign trade of the individual products in the last year of the plan period (1970) are generally represented by seven "standard" variables:

1. The *production* of the product in the last plan year, with fixed capital that was already in operation at the beginning of the plan period and remains unchanged until the end. This requires only the maintenance of the old fixed capital during the plan period.

2. The *production* of the product in the last plan year, with fixed capital that was already in operation at the beginning of the plan period, but whose technology is changed by reconstruction in the course of that period. This requires not only maintenance but also technical reconstruction of the old fixed capital during the plan period.

The value of variables 1 and 2 determines the fate of old fixed capital. If variable 1 has a positive value, the old fixed capital must be maintained without any change; if variable 2 has a positive value, a reconstruction of the old fixed capital must be carried out. If the value of both these variables is zero, the old fixed capital must be dismantled.

3. The *production* of the product in the last plan year, with fixed capital invested in the course of the plan period.

In the case of numerous products, several variables – such as 3.1, 3.2, etc. – figure instead of a single one, to represent the different technological variants of new plants.

The computation of variables 1, 2, and 3 determines the pattern of *production* in the last plan year. At the same time it also determines the pattern of *gross investment* (maintenance, reconstruction, and new investment) during the plan period. This again involves the elaboration of basic estimates of *technical development* because the proportions of variables 1, 2, and 3 (and also those of 3.1, 3.2, etc., representing the technological variants) will to a great extent determine the technological pattern of production.

4. *Imports* of the product from socialist markets.

5. *Imports* of the product from capitalist markets.

6. *Exports* of the product to socialist markets.

7. *Exports* of the product to capitalist markets.

As may be seen, the program computes not only the volume of foreign trade but also its breakdown by major markets.

To sum up: *the economy-wide program yields a complex production, investment, technical development, and foreign trade plan with respect to 491 priority products.*

To measure volume, the physical units generally employed in planning were used wherever possible (with 406 of a total of 491 products). Only for products that cannot be measured in this way were value terms used, based on producer prices.

Not all of the 491 products have all seven standard variables. With some products the old fixed capital does not lend itself to technical reconstruction. Others cannot be used for all four foreign trade activities.

The 491 products do not represent the whole of social production. The *external sphere*, which is not represented by any variable in the model, falls into two parts. The first of these comprises the sectors not covered by the model, such as metallurgy and transport.[3] The 1965 input-output table of the National Bureau of Statistics divided the national economy into 74 sectors.[4] Of these, only 52 sectors appear with their priority products in the model described here.

The second part of the external sphere comprises the residual, nonpriority activities – if any – of the sectors that appear in the model with their *priority* products only.

The omitted sectors could, in principle, be built into any future model without great difficulty. The nonpriority activities of the sectors covered by the present model, on the other hand, involve some difficult problems that will be dealt with in the second part of the paper.

When defining the model's variables, it was assumed that the individual products have the same homogeneous properties, whether domestically produced or imported, or whether intended for domestic use or export. This assumption is universally made in the construction of models and in planning based on product balances, the traditional method adopted in socialist countries.

When defining the model's variables, only investment projects not extending beyond the plan period were taken into account.

The resources serving the purposes of so-called "overlapping" investments were deducted from the resources available for allocation by the model.

The model has a total of 2,424 activity variables. The figure does not include the so-called auxiliary (slack, surplus, and artificial) variables.

1.2 ECONOMIC CONTENT OF THE CONSTRAINTS

The system of constraints may be considered from two aspects of classification. Let us first group the constraints according to their economic contents.

The constraints are presented in Table 1, which gives the number of constraints belonging to each constraint group, the unit of measurement used, and the time period (i.e., whether the data presented refer to the last plan year or to the entire plan period).

1. *Internal product balances.* These coordinate in the model the outputs represented by variables (production, imports) with the inputs represented by variables (productive input, exports, additional consumption) and the inputs given as constant ("compulsory" private and public consumption, productive input into the external sphere, investment and reconstruction requirements in the last plan year, changes in inventories).

An internal product balance applies to most products. In some cases, however, the balance of input and output is expressed in a somewhat different form. This happens, for example, when producers use a different breakdown for their output than users do for their input. This accounts for the difference between the number of products and that of internal product balances.

2. *External material balances.* These limit the model's production variables in the input of *external materials.* The latter are products and services that are not the output of the model's production variables and whose total available quantity is given as constant. (For some external materials, the import of the material in question from one or another market is treated as a vari-

A SURVEY

1	Internal product balances	Physical units or forints	Last plan year
2	External product balances	Physical units or forints	Last plan year
3.1	Wage-fund constraints	Forints	Last plan year
3.2	Live labor constraints	Persons	Last plan year
4	Land constraints	Cadastral acres	Last plan year
5	Capacity constraints	As constraint group 1	Last plan year
6	Special technological proportionalities	As constraint group 1	Last plan year
7.1 - 7.3	Gross investment quotas: construction; domestic machinery	Forints	Whole plan period
7.4 - 7.5	Gross investment quotas: imported machinery	Rubles, dollars	Whole plan period
8.1	Export and import constraints by products	As constraint group 1	Last plan year
8.2	Constraint of machinery exports	Rubles, dollars	Last plan year
9	Foreign exchange balances	Rubles, dollars	Last plan year

Total

Table I

OF THE CONSTRAINTS

43	56	405	504
10	15	50	75
3	-	-	3
4	-	15	19
-	7	3	10
-	10	719	729
-	-	45	45
3	-	-	3
2	-	-	2
-	-	661	661
-	2	-	2
2	-	-	2
67	90	1,898	2,055

able, and only the quantity available from other imports and from domestic production is given as constant.)

3. *Live labor constraints.* These limit the total input of the wage fund and labor. Separate constraints limit the input of male labor as well as the flow of labor between the agricultural and non-agricultural spheres.

4. *The constraints of land.* In the model there are six types of land available to the collective agricultural plants: loose, semihard, and hard ground, all three cultivated by dry or irrigation technology. In addition to these six agronomical constraints, a seventh land constraint is socioeconomic – the household plot.

5. *Capacity constraints.* Production based on unchanging technology is limited by the available old capacities. The possibilities of reconstruction and plant enlargement are generally also limited. In special circumstances, construction of new plants is also bounded from above.

In the four sectors of plant cultivation, special constraints were prescribed for the stock available in 1970 of the ten most important machine types.

6. *Special technological proportionalities.* These usually prescribe the technological ratios between the various production variables within some sphere of production (e.g., mixing conditions in the chemical industry).

7. *The constraints of investment resources.* These limit the amounts available for gross investment – maintenance, reconstruction, and the construction of new plant – over the whole plan period. The constraints are given in aggregate form, as well as separately, for the input of construction, domestically produced machinery, and imports of machinery from both socialist and capitalist countries.

8. *Foreign trade constraints.* All export variables are constrained with respect to every product, in order to express the foreign buyers' limited propensity to import. Similarly, individual constraints by product were prescribed for every socialist import variable, in order to express the sellers' limited propensity to ex-

port. On the other hand, no individual constraints were put on capitalist import variables because here there is practically no upper bound to the propensity to sell, the constraint being represented by our own ability to buy. (The latter is expressed by the foreign exchange balances.)

An upper bound was also prescribed for the total export of machinery, to represent the difficulties of market expansion.

9. *Foreign exchange balances.* Separate foreign exchange balances were given for trade in socialist and capitalist markets. Export earnings constitute positive items in the balance. The negative items are made up of expenditures connected with the model's import variables (expenditure on priority, competitive imports) and of import expenditures connected with the input by the model's production variables of nonpriority products that do not figure among the 491 products of the model (expenditure on nonpriority, noncompetitive imports).

The model has a total of 2,055 constraints.

As in defining variables and products, in the construction of the system of constraints we followed the index system of the National Planning Office and the ministries. This applies to constraint groups 1 and 2, which are closely related to the traditional system [5] of product balances; to constraint group 3, which contains part of the traditional manpower balance system; to group 7, which follows the traditional breakdown of investment quotas. Because of special characteristics of the model, the structure of constraint group 9 differs considerably from the traditional pattern of the balances of payment and of foreign trade; the differences can, however, be explained numerically.

Constraint groups 4, 5, 6, and 8 do not appear in direct form in the index system of traditional planning, although in practice the planners will try to take into account the limits set by the area of land, the old capacities, and the possibilities of selling to and buying from foreign markets.

In summary, as regards the number of variables and constraints, the model describes the relationships of the five-year plan in a linear equation system containing 2,055 equations and 4,479 variables (with the auxiliary variables included). Linear programming has already been used for economic planning in such countries as France, India, and Czechoslovakia. According to available information, *the Hungarian model is the largest and most detailed of the economy-wide planning models known so far.*

1.3 "LEVELS" OF THE SYSTEM OF CONSTRAINTS

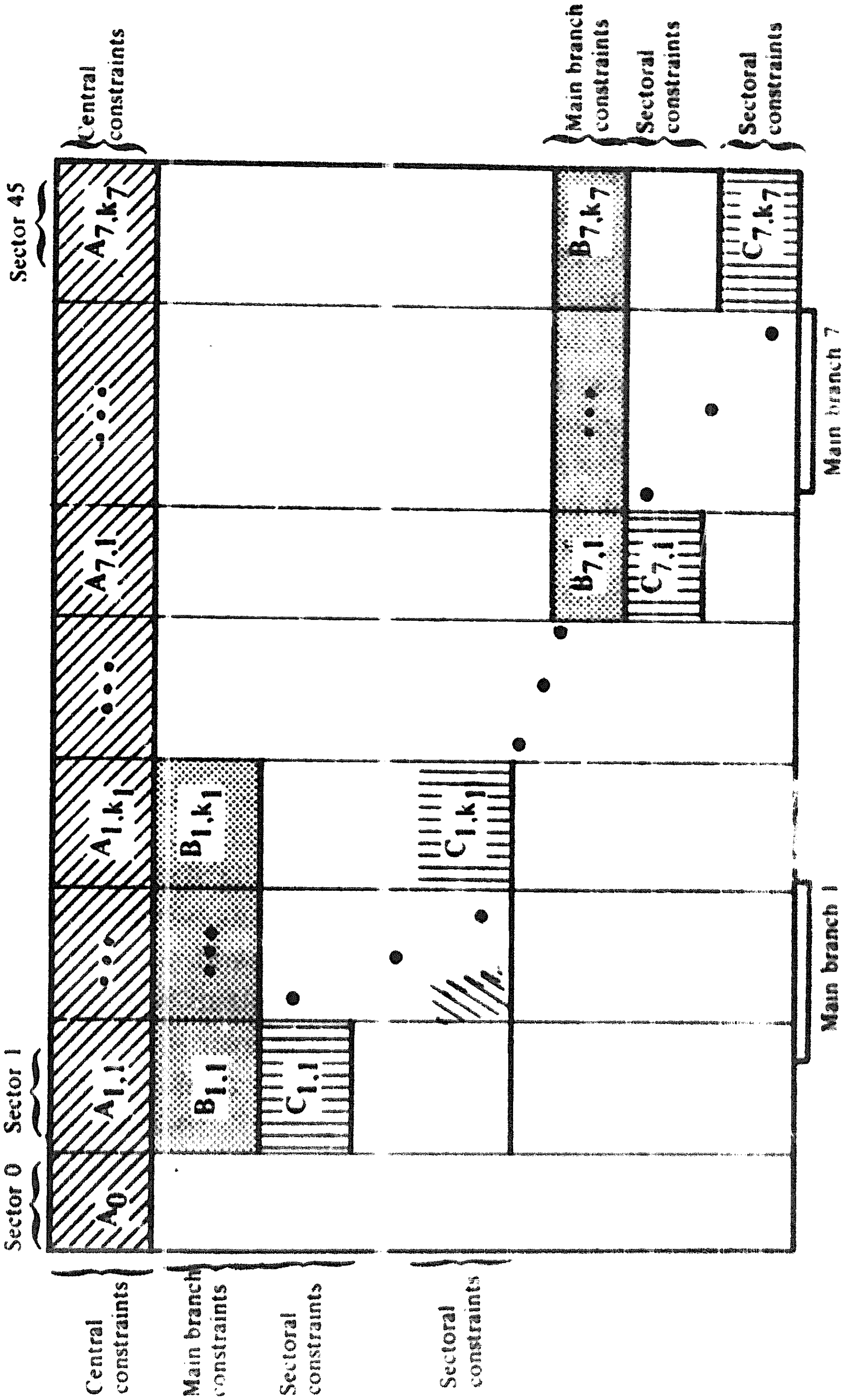
The constraints were classified above according to their economic content. Let us now turn to another type of classification, by "levels." To facilitate understanding, the schema of the matrix of coefficients of the equation system is given in Figure 1.[6] We will refer to this schema in explaining the breakdown of the model.

The model was divided into 46 *sectors*, and every activity variable was assigned uniquely to one of the sectors. The breakdown is not arbitrary, but reflects real economic content. Sectors 1 to 45 each represent a sphere responsible for the production and foreign trade of a definite group of priority products. The productive activities of the sector generally represent one or several administrative units or institutions (industrial directorates, trusts, enterprises, etc.). Thus, the sector of the paper industry, for example, represents the Paper Industry Enterprise, the sector of the automobile and tractor industry represents the Motor and Tractor Trust, and so on. With each productive activity, the corresponding export and import activities were assigned to the same sector. These are usually handled in practice by some other institution, by one or several foreign trading companies. The sector thus unites, so to speak, the planning work of the institutions responsible for the production and foreign trade of the products belonging to its sphere.

To sector 0 were assigned import variables of the external materials needed in several sectors. Accordingly, this "sector of external material imports" does not represent any real institution.

Figure 1

THREE-LEVEL ARRANGEMENT OF THE MATRIX OF COEFFICIENTS



In the schema, the matrix of coefficients is vertically divided by the 46 sectors into 46 narrow column clusters.

The model was divided into seven *main branches*. Sectors 1 to 45 were each uniquely assigned to one of the main branches; sector 0 was assigned to none. Like the breakdown by sectors, that by main branches is not arbitrary; it follows the pattern of the economic ministries. Each main branch represents either a whole ministry (e.g., the light industry or the building industry) or a section of a ministry that is in reality to some extent independent of the rest (e.g., the chemical industry from the power industry and mining, the food industry from agriculture, etc.).

In the schema, the seven main branches divide vertically, into seven broad column clusters, the part of the coefficient matrix that follows the sector marked 0.

After the vertical division, let us now deal with the horizontal division. In the subsequent description the sectors will be given a double suffix. The first suffix refers to the serial number of the main branch: i = 1, . . . ,7. The second suffix gives the serial number of the sector within the main branch:

$$j = 1, \ldots, k_i \qquad \sum_{i=1}^{7} k_i = 45.$$

The constraints are grouped into three principal categories.

Sectoral constraints. These contain coefficients other than zero exclusively within the sector. Some examples are:

capacity constraints of production in old plants with unchanged technology or reconstructed equipment;

export and import constraints of individual products; and

internal balances of products that do not constitute an input for another sector of the model.

The sectoral constraints regulate the sector's "internal affairs" and its direct relations with the "outside world" – that beyond the scope of the model.

In the schema, the blocks of the sectoral constraints are horizontally striped and marked $C_{1,1}, \ldots, C_{1,k_1}, C_{7,1}, \ldots, C_{7,k_7}$.

The model contains a total of 1,898 sectoral constraints.

Main branch constraints. These contain coefficients other than zero in several sectors, all of which belong, however, to the same main branch. Some examples are:

the common land, labor, and machinery constraints of agriculture;

the common export constraints of the engineering industries; and

internal balances of products that constitute the output of a sector and the input of another or several other sectors within a main branch, but of no sector in the model outside the main branch concerned (e.g., various chemical products within the main branch of the chemical industry).

The main branch constraints regulate the main branch's "internal affairs," the mutual relations of its sectors, as well as its direct relations with the "outside world" – that beyond the scope of the model.

In the schema, the blocks of the main branch constraints are vertically striped and marked $B_{1,1}, \ldots, B_{1,k_1}, \ldots, B_{7,k_7}$.

The model contains a total of 90 main branch constraints.

Central constraints. These contain coefficients other than zero in several main branches. Some examples are:

constraints of live labor in the national economy as a whole;

investment quotas; and

internal balances of products that are the output of a sector in one of the main branches, but are also used as input in other main branches (e.g., electric energy, wrapping paper, etc.).

The central constraints regulate the "common affairs" of the main branches.

In the schema, the blocks of the central constraints are diagonally striped and marked A_0 $A_{1,1}, \ldots A_{1,k_1}, \ldots, A_{7,1}, \ldots, A_{7,k_7}$

The model contains a total of 67 central constraints.

In the terminology used here the attributes "sectoral," "main branch," and "central" indicate the "level" of the constraint.

The present economy-wide model is, in its final form, a three-

level one.

In Table 1, the number of constraints falling into the different levels is given for each constraint group, with classification according to economic content.

The multi-level structure is a particularly important characteristic of the present model. This property differentiates it most clearly from the mathematical programming models used for economy-wide planning purposes in the past, either in this country or abroad.

1.4 THE OBJECTIVE FUNCTIONS

In the series of computations surveyed here, several types of objective function were alternatively employed.

1. The maximization of additional consumption over and above "compulsory" private consumption. The composition of additional consumption was given with several product pattern variants.

2. The maximization of the surplus of the balance of capitalist foreign exchange.

3. The maximization of the surplus of the balance of socialist foreign exchange.

4. The minimization of total gross investment.

5. The minimization of live labor input (with two variants – minimization of manpower and minimization of the wage fund).

6. In some subsidiary calculations: the minimization of prime costs at current prices and of costs at calculative prices.

In some computations (e.g., in parametric programming), certain combinations of the above objectives were prescribed. Thus, the two types of foreign exchange balance were optimized jointly, etc.

2. A SYSTEM OF MODELS

2.1 THREE MODEL TYPES – THE UNION OF MODELS

Because of the special multi-level structure of the model described here, it is possible not only to employ it as a single large economy-wide model but also to carry out independent calculations by means of its *blocks* suitably separated from one another.

Three different *model types* have been worked out.

The sector model. This is used exclusively for programming the activities of a single sector: the j^{th} sector of the i^{th} main branch. The programming problem may be described as follows:

$$
\begin{aligned}
A_{ij}\ x_{ij} &= g^{(A)}_{ij}, \\
B_{ij}\ x_{ij} &= g^{(B)}_{ij}, \\
C_{ij}\ x_{ij} &= g^{(C)}_{ij}, \\
x_{ij} &\geq 0, \\
p'_{ij}\ x_{ij} &\rightarrow \max,
\end{aligned}
\tag{1}
$$

where

x_{ij} = the program vector;

$[g^{(A)}_{ij}, g^{(B)}_{ij}, g^{(C)}_{ij}]$ = the sector's constraint vector partitioned according to the central, main branch, and sectoral constraints;

p'_{ij} = the vector of the objective function coefficients.

When constructing sector model (1), the sector is separated from the whole of the national economy, to which (and, within which, to the other sector models) it remains connected by the appropriately determined vectors $g^{(A)}_{ij}$ and $g^{(B)}_{ij}$, the constraint constants belonging to the central and main branch constraints.

As normally dimensioned, the sector models contain 30 to 80 constraints and 60 to 100 activity variables.

The main branch model. This unites all k_i sectors of the i^{th} main branch. The programming problem may be defined as follows:

$$\sum_{j=1}^{k_i} A_{ij} x_{ij} = g_i^{(A)} ,$$

$$\sum_{j=1}^{k_i} B_{ij} x_{ij} = g_i^{(B)} , \qquad (2)$$

$$C_{ij} x_{ij} = g_{ij}^{(C)} , \qquad j = 1, \ldots, k_i$$

$$x_{ij} \geqq 0 , \qquad j = 1, \ldots, k_i$$

$$\sum_{j=1}^{k_i} p_{ij}' x_{ij} \to \max ,$$

where

$[g_i^{(A)} , g_i^{(B)}]$ = the constraint vector of the main branch, partitioned according to the central and main branch constraints.

When constructing main branch model (2), all sectors within the main branch are linked, but the main branch itself is separated from the national economy as a whole. It remains connected with the rest of the economy (and, within the latter, with the other main branch models) exclusively by the appropriately defined vector $g_i^{(A)}$, the constraint constants belonging to the central constraints.

As normally dimensioned, the main branch models contain 150 to 300 constraints and 300 to 500 activity variables.

The economy-wide model. This unites all sector models. The programming problem is the following:

$$A_0 x_0 + \sum_{i=1}^{7} \sum_{j=1}^{k_i} A_{ij} x_{ij} = g^{(A)}$$

$$\sum_{j=1}^{k_i} B_{ij} x_{ij} = g_i^{(B)} \quad , i = 1, \ldots, 7$$

$$C_{ij} x_{ij} = g_{ij}^{(C)} \quad , \tag{3}$$

$$i = 1, \ldots, 7, \ j = 1, \ldots, k_i$$

$$x_{ij} \geqq 0 \ ,$$

$$i = 1, \ldots, 7; \ j = 1, \ldots, k_i$$

$$\sum_{i=1}^{7} \sum_{j=1}^{k_i} p'_{ij} x_{ij} \rightarrow \max,$$

where

$g^{(A)}$ = the national economy's constraint vector ordered to the central constraints.

As mentioned, economy-wide model (3) contains 2,055 constraints and 2,4[illegible]4 activity variables.

The three model types differ from each other in the breadth of the sphere they cover – a single sector, a main branch composed of 3 to 11 sectors, or the whole of the economy with all its 46 sectors.

2.2 DECOMPOSITION – THE CONNECTION OF MODELS

Two principal methods for solving the programming problem offered themselves.

1. *The direct solution.* The problem may be solved directly, by means of an appropriate algorithm (e.g., the simplex method). This method has been applied in every case to the sector models. With the main branch and economy-wide models, however, this was not possible with the facilities available in Hungary.[7]

2. *The decomposition method.* Among several decomposition methods known are, first, the Dantzig-Wolfe[8] and those formulated on the basis of the theory of games.[9]

Without attempting full description or comparison of the various decomposition methods, let us survey some characteristic common features, especially as they relate to the present special problem. For simplicity's sake, the exposition will be based on a two-level model instead of a three-level one.

In the direct solution, calculations are carried out simultaneously with the whole system of equations. (In our case certain simultaneous calculations would be carried out with 2,055 equations.) With the application of the decomposition method, on the other hand, it becomes possible to reduce considerably the size of the equation systems handled at a time. These smaller equation systems may be classed into two main categories: *higher- and lower-level equation systems,* and computations carried out with them may be termed *higher- and lower-level computations.*

The four main criteria of the decomposition methods are:

1. Instead of solving a single large equation system in a single calculation, [illegible] small[illegible] equation systems must be solve[illegible]

eral times.

2. The higher-level computations are more aggregate in character while the lower-level computations are more disaggregate and more detailed.

3. The method is iterative. In every iteration both higher-level and lower-level computations are carried out.

4. In every iteration, the higher-level computations yield new information relevant to the lower-level ones, and vice versa. A two-way flow of information occurs, providing a basis for repeated iteration on both levels.

The decomposition methods differ from each other in the realization of the above four criteria:

(a) the principles of the decomposition of the model and in the content and size of the higher- and lower-level equation systems;

(b) the degree and character of aggregation and disaggregation on the higher and lower levels;

(c) the character of the computations to be carried out on the higher and lower levels;

(d) the character of information flowing between the two levels.

Experience shows that the known methods of decomposition are extremely slow. With the present facilities of computing techniques in Hungary, therefore, they could not be employed in the project. Instead, an *approximation decomposition method* was worked out.

2.3 THE APPROXIMATION METHOD[10]

The basic concepts of the approximation method were taken from the Dantzig-Wolfe algorithm, of which it may be considered a naive-heuristic variant. Given this algorithm, the method will be described in outline for the sake of simplicity, as a two- instead of a three-level problem.

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The structure of the *large problem* before decomposition is:

$$
\begin{aligned}
A_1x_1 + A_2x_2 + \ldots + A_nx_n &\leqq b_0 \\
B_1x_1 \qquad\qquad\qquad\qquad &\leqq b_1 \\
B_2x_2 \qquad\qquad\qquad &\leqq b_2 \\
\ddots \qquad\qquad &\vdots \\
B_nx_n &\leqq b_n \\
x_1 \geqq 0, x_2 \geqq 0, \ldots, x_n \geqq 0 & \\
c_1'x_1 + c_2'x_2 + \ldots + c_n'x_n \rightarrow \max. &
\end{aligned}
\tag{4}
$$

Let us call the problem below the k^{th} sector problem of the i^{th} sector:

$$
\begin{aligned}
A_ix_i &\leqq u_i^{(k)} \\
B_ix_i &\leqq b_i \\
x_i &\geqq 0 \\
g_i'^{(k)}x_i &\rightarrow \max.
\end{aligned}
\tag{5}
$$

Let $x_i^{(k)}$ denote the optimum solution of problem (5). Before starting the computation, a feasible solution of problem (4) is known; let us call this the *comparative program* and denote $x_i^{(1)}$. In the experimental computation outlined here, the official program based on nonmathematical methods was considered the comparative program.

Let us call this the *sector optimum program* and denote $x_i^{(2)}$ as the optimum solution of a sector problem of type (5), where

$$u_i^{(2)} = A_i x^{(1)}, \quad g_i^{(2)} = c_i. \tag{6}$$

In our experience, the objective function value of the sector-optimum program was in every case higher than that of the comparative program.

For our procedure, let us generate, using sector program $x_i^{(k)}$, the vector $t_i^{(k)}$, the k^{th} *plan proposal* and real number $\gamma_i^{(k)}$, the k^{th} *objective function contribution:*

$$t_i^{(k)} = A_i x_i^{(k)}, \quad \gamma_i^{(k)} = c_i' x_i^{(k)}. \tag{7}$$

The approximation method is iterative. In each iteration, one or several plan proposals are drawn up in every sector for the central computations. The earlier generated plan proposals are not disregarded in the course of the later iterations, but continuously accumulated. Thus, in the z^{th} iteration, a total of $\sum_{i=1}^{n} K_i(z)$ plan proposals are available, including comparative plan proposal No. 1 and the sector-optimum plan proposal No. 2.

In each iteration – hence, in the z^{th} also – a *central* problem must be solved

$$\begin{array}{llll} T_1(z)y_1(z) + T_2(z)y_2(z) + \ldots + T_n(z)y_n(z) & = b_0 & \\ 1'y_1(z) & = 1 & \\ \qquad 1'y_2(z) & = 1 & \quad (8) \\ \qquad\qquad \ddots & \vdots & \\ \qquad\qquad\qquad 1'y_n(z) & = 1 & \\ y_1 \geqq 0,\ y_2 \geqq 0, \ldots, y_n \geqq 0 & & \\ \gamma_1'(z)y_1(z) + \gamma_2'(z)y_2(z) + \ldots + \gamma_n'(z)y_n(z) \to \max, & & \end{array}$$

where $T_i(z)$ is a matrix composed of a total of $K_i(z)$ plan proposals as column vectors; $\gamma_i'(z)$ is a row vector composed of the same number of objective function contributions. *Weight vector* $y(z) = [y_1(z), y_2(z), \ldots, y_n(z)]$ constitutes the variable of the central problem and indicates the degree of acceptance of the plan proposals. As we have seen, for each sector the sum total of the weights is one.

The exact Dantzig-Wolfe algorithm gives a strict rule for the procedure of composing a new plan proposal for the central problem of the $(z + 1)^{th}$ iteration, using the dual solution of the central problem of the z^{th} iteration. The approximation method regulates only plan proposals Nos. 1 and 2, while for composing plan proposals Nos. 3, 4, . . . ,$K_i(z)$ there is no strict rule. New plan proposals can be formed in a deliberate manner, with the suitable choice of the pair of vectors $[u_i^{(k)}, g_i^{(k)}]$. Some general viewpoints of composing the plan proposals are:

A. In case of the scarcity of one of the central resources:

A.1. the upper bound of the utilization of the scarce resource in question can be reduced when determining $u_k^{(k)}$, or

A.2. the minimization of the utilization of the scarce resource in question can be prescribed as an objective function.

B. In a redundance of one of the central resources, the upper bound of its utilization can be raised when determining $u_i^{(k)}$, with a possible simultaneous reduction of the upper bound of another scarce resource. The sector is thus compelled to carry out substitution.

C. The center may carry out intersectoral comparisons. Thus, it may compare the shadow prices obtained for constraint vectors u_i in the course of sectoral programming, and may, on the basis of the comparison, suggest a wider resource bound for the sectors ensuring more efficient utilization, and a narrower one for the less efficient sectors.

Viewpoints A, B, and C may be combined. Furthermore, either constraint vector $u_i^{(k)}$ or objective function coefficient vector $g_i^{(k)}$ can be given in parametric form. In this manner a program series to provide a basis for several plan proposals can be obtained in a single parametric programming computation.

After every iteration, the improvement achieved as compared with the comparative program's objective function value may be computed. On consideration of the improvement, the question of terminating the computation will be deliberately decided. Let us suppose that the computation is terminated in the Z^{th} iteration. Then the improved program x(Z) is computed:

$$x(Z) = X(Z)\ y(Z)^* \tag{9}$$

where X(Z) is the matrix of the programs providing a basis for the plan proposals accumulated up to the Z^{th} iteration, and $y(Z)^*$ is the optimum solution of the central problem of the Z^{th} iteration.

It is possible to prove the following properties of the procedure:

1. The improved program x(Z) is a feasible solution of the original large problem (4).

2. The objective function value of the improved program x(Z) is definitely more advantageous than that of the comparative program.

3. The procedure is monotonous; the objective function value of the improved program obtained in the z^{th} iteration is not less advantageous than that obtained in the $z - 1^{th}$ iteration.

Although they cannot be mathematically proven, practical experience has shown that the procedure also has other advantages. (Precisely because the determination of vector pairs $[u_i^{(k)}, g_i^{(k)}]$ is not algorithmic but heuristic, the efficiency is not guaranteed but will depend on the planners' ability.) The following properties should be pointed out:

The simplex-type algorithms of linear programming proceed from extreme point to extreme point of the set of feasible programs. In the course of this procedure we usually start from a point that is absurd from the planners' point of view and uninterpretable (or, at least, highly disadvantageous); we reach the favorable extreme points only after a large number of iterations.

The approximation method, on the other hand, starts from an interior point of the set and – although no guarantee exists that the optimum extreme point will be reached – the interior point reached in the first iteration will be comparatively sensible, non-absurd, and not particularly unfavorable. Since the central problem of the first iteration contains plan proposals Nos. 1 and 2, the point reached will not be worse than the latter.

When determining vector pairs $[u_i^{(k)}, g_i^{(k)}]$, the planners may draw extensively on the experiences gained in the course of the computations of the primal and dual solutions of both the central and the sector-level problems. Moreover, they may utilize the information material not built into the model but otherwise available, their own planning routine, their knowledge of the actual scarcities and redundancies in the economy and of the possibilities of substitution that promise higher efficiency, etc.

A further advantage is that each plan proposal has a characteristic profile and characteristic economic content (e.g., "proposal with maximum investment savings," "proposal ensuring maximum technical development," etc.). The weight vector y(Z) thus acquires a definite economic content by defining the relative weights of the various possible policies of sector development.

The procedure may be particularly efficient when the aim is not approximation of the solution of a single large-scale problem, but computation of variants with different central constraint vectors b_0 and different objective functions. In such cases, it will be possible to "blend" the approximate solution of different large problems from the same set of plan proposals.

The approximation method was resorted to as a consequence of technical computing difficulties; it would therefore not be appropriate to make a virtue of this necessity. Because of the uncertainty of the initial data, however, exact optimality, in the mathematical sense of the term, is not very important in planning. The approximation method therefore appears acceptable for practical purposes for the time being.

2.4 "SYSTEM" – IN WHAT RESPECT?

The use of the term "system" (in the system-theoretical, cybernetical sense) will be warranted if we are dealing with a structure or network of *interconnected* elements. In this sense, the methodology of the project surveyed here has resulted in evolving a *model system* whose elements are the various models.

The relationship between the elements can be approached from two aspects. One aspect was dealt with in section 2.1, i.e., the submodels can be united with one another. With the combination of submodels, "model-building" can be done; for example, a "two-story" building can be formed by uniting a group of sector models into a main-branch model or all 46 sector models directly into an economy-wide model. Or it is possible to form a "three-story" building by uniting the seven main-branch models (with sector model 0) into an economy-wide model.

The other aspect was discussed in sections 2.2 and 2.3. In the application of exact or approximation decomposition methods, the submodels are not united but connected, *linked* to each other *by means of information flows* between lower- and higher-level equation systems worked out in accordance with the concrete principles of the decomposition method.

Both the union of the submodels and their linking by means of information flows became possible because *all models are constructed on the basis of a uniform index system.* Quantification was always based on strictly uniform nomenclatures, statistical definitions, and classification.

An important property of the model system is *the unique communication between the higher- and lower-level models and computations.* The higher-level computations may yield, among others, aggregate economic indices, but we can always make detailed production, investment, and foreign trade programs correspond uniquely to them. Conversely, we can always uniquely derive from the detailed production, investment, and foreign trade programs aggregates established on the sectoral, main-branch, or economy level.

Again, we have a characteristic feature of the methodology of the project that distinguished it from the mathematical programming models used earlier for planning. The projects known from Hungarian and foreign literature were all based on individual and independent models. *Now, for the first time, an interdependent system of planning models, a network of aggregate and disaggregate models, a combined hierarchical structure of higher- and lower-level models can be worked out successfully.*

3. THE DATA OF THE MODEL

The data and parameters employed in the calculations that provide a foundation for economic decisions may be determined in various ways. Let us first describe three pure cases and deal with their various combinations later.

1. The phenomenon to be numerically described by the parameter is subjected to full-scale observation. This is the case, for example, when the inputs and outputs of a strictly defined economic unit (an enterprise, an economic branch, etc.) are determined on the basis of full-scale statistical observation.

In this case the economist carrying out the economic calculation will obtain the parameter directly from the *statistician* summarizing the data yielded by full-scale observation, and will use it in his economic calculation without transformation.

Let us call this method the *simple economic-statistical method.*

2. However, certain statistical observations available do not directly characterize the phenomenon to be described by the parameter of the economic calculation, but allow only indirect inferences. In indirect inferences, the tools of mathematical statistics are employed. For example, a trend calculated on a time series is extrapolated into the future. Or a confidence interval estimate is given, based on the mathematical-statistical analysis of data distribution. Or again, an estimation of the parameter is worked out by determining an appropriate average value on the

basis of representative sampling.

The economist carrying out the economic calculation obtains the data not directly from those carrying out and summarizing the observations, but from the *mathematical statistician* who processes their results.

Let us call this method the *mathematical statistical method* (in Western terminology, the *econometric method*).

3. Such available observations as statistical data, technical or commercial information, etc., do not directly characterize the phenomenon to be described by the parameter of the economic calculation, but allow indirect inferences that are drawn without using the tools of mathematical statistics, in a basically intuitive manner For example, an engineer who knows precisely the numerical characteristics of present technology and also has information concerning future technology knows the differences between the two technologies in qualitative, technical terms, and he may also have numerical information obtained from the literature or from personal consultations. Relying on these, he will estimate the numerical characteristics of the new technology. A foreign trade expert who knows the exact market situation, past price trends, and relevant sales data has an idea of the intentions of the buyers concerning the future, an idea formed in personal contacts. Using this information, he will estimate future price formation and sales possibilities.

The estimates thus given may be primitive, inexpert, irresponsible; but they may also be based on technical calculations and formulas, on careful collection of information, worked out with expert knowledge and a sense of responsibility. In the latter case they will be more reliable, but still not "exact." Because intuition still operates, the process of transforming the acquired information into an estimate will not lend itself to mathematical formulation, to description in exact terms.

In this case the economist carrying out the economic calculation obtains his data from *the engineer, the foreign trade expert, the specialist.*

Let us call this method that of *expert estimation.*

The differences between the three methods are:

In the first case the data observed are built *directly* into the economic calculation; in the second and third they are used indirectly. Transformation of the observations takes place in the second case *on the basis of mathematically formalized rules*, by means of mathematical statistical methods, and in the third case *without formalized rules*, on the basis of expert knowledge.

In practice the three methods are frequently combined. For example, the data obtained by means of the first or second method may not be used directly in the economic calculation, but *corrected* first by expert estimation to make them express more adequately the differences between the phenomena observed in the past and those expected in the future. Or the third method may be basically applied, but with the expert estimation making extensive use of full-scale statistical observations and information obtained by mathematical statistical tools (trend computations, averages, functions quantified through econometric methods, etc.).

Most economic decisions are, in actual practice, reached through the third method: calculations based on expert estimation, frequently supplemented and combined with the first and the second method as described. This applies particularly to investment decisions, under both the socialist planned economy and the capitalist market economy. As a matter of fact, such decisions make extensive use of calculations relating to new, untested technologies, as well as to price forecasts and future marketing possibilities.

Part of the mathematical models used in planning is based essentially on the first, simple economic-statistical method, e.g., the input-output tables quantified on the basis of full-scale statistical observation.

Another part of the planning models is based essentially on the second mathematical statistical method, e.g., the econometric macromodels (the Dutch planning models, the Klein-Goldberger model worked out in the USA, the Hungarian M-1, M-2 models, and others).

Quantification of the model we describe was carried out essentially by the third method: expert estimation. This important characteristic distinguishes it from a number of other planning models constructed in Hungary and abroad. The structure of the model – especially the great number of production, investment and foreign trade variables – makes it imperative to rely to a great extent on the estimates of technical and foreign trade experts.

Only a comparatively minor proportion of the data was based on simple economic-statistical observation; in the main, that method was used when determining the parameters of the variables of type 1 (the operation of old plants with unchanged technology). In a further data group, mathematical statistical methods were used – in calculations for trends in world market prices, in working out the demand functions required to determine the pattern of consumption, etc. But even here the data obtained had to be corrected on the basis of expert estimates to make them suitable for practical application.

Some of the data used were taken from the official documentary material of traditional planning work based on nonmathematical methods, in unchanged form or with corrections carried out in cooperation with experts. Other data, which could not be found in official documentary material, were estimated by experts, and the specialists were usually engaged in similar tasks in connection with official planning projects. Thus, the quality of the estimates was similar in both cases.

We will revert to the supply of information in traditional planning in the second paper. The information basis of the model described here was essentially the same as that of the traditional plans of the investment calculations based on nonmathematical methods; in part, the two methods share the same basis. *The information material that would otherwise extensively influence the medium-range economic decisions but that is usually utilized in scattered form, without strict logical schematization, is united in* *the present model according to a uniform classification, in consistent and logical arrangement.*

* * *

In conclusion, the first experimental computation series of economy-wide programming has *proven the possibility of constructing a multi-level model system.* This is significant but not sufficient. These questions must also be answered: How can the method be employed in actual planning work? What are its practical purpose and role? What are the shortcomings of the first experiment? What are the conditions of its systematic practical application? We answer these questions in the second part of the paper.

PART II

PRACTICAL APPLICATION OF THE MULTI-LEVEL PROGRAMMING MODEL OF THE NATIONAL ECONOMY

The main purpose of the experiment was to test a new method in mathematical programming.[11] What possibilities of future application does the new method offer? In the course of the exposition not only will concrete realization of the first experiment be kept in view, but also the task of further developing the model. Similarly, treatment of the model's "environment," of traditional planning methods, and of the relationship between mathematical and nonmathematical methods will not be confined to describing the present situation; modifications expected -- or desired -- will also be mentioned, although utopian ideas will be avoided. We will consider only those changes that appear possible under the given objective conditions and whose realization depends fundamentally on being implemented.

4. THE MODEL'S SPHERE OF ACTION

The multi-level economy-wide programming model embraces a broad sphere of the economy and can simultaneously determine several thousand plan indices. The sphere of action of the model may still be broadened by further development. Neither this nor any other model, however, can perform all functions of planning. It is impossible to work out any "super-model" into which the ready data may simply be fed and a complete national economic plan retrieved.

4.1 PROGNOSIS AND PRESELECTION

The model described here belongs to the family of *programming* models. The members of the model family have the common characteristic of a clear distinction between the *given conditions* and the *possibility of choice*. When the structure and numerical material of a programming model are determined, one also determines the given conditions (expressed by the constraints) and, within them, the possibilities of choice (represented by the variables). Instead of a single computation a whole calculation series can also be carried out, repeatedly changing either the constraints or the variables. It remains valid for every member of the series, however, that what can and cannot be considered a variable has been determined at the beginning of the calculation.

It follows that *the planning functions of programming are complemented and preceded by two other functions: prognosis and preselection.*

1. *Prognosis* gives an answer to questions about the future. What can be expected with certainty to materialize in the event, independently of the resolutions of the decision-makers? Where can the latter interfere at all, and to what extent? What are the limits of interference? What are the expected consequences of alternative economic activities? As can be seen, part of the prognosis is "unconditional" and part dependent on certain "condi-

tions," because they deal with events and processes that would come about only if certain conditions were fulfilled.

Prognosis may be based on primitive forecasting; it may, however, be prepared also with more circumspection, with a careful analysis and mathematical statistical examination of domestic time series, or on the basis of extensive international comparisons, or with special prognostic models.

2. In reality, an infinite variety of economic activities is possible. From among them, a finite number is chosen by preselection; these become represented in the model by the variables. (Some 2,500 were chosen for the first experimental model of the project described here.)

Preselection may take place arbitrarily; important alternatives may be left out of consideration because of inadequate information or subjective bias. On the other hand, it may also take place on the basis of an extensive collection of information and by means of special models worked out for the purpose.

The reliability of the programming model will depend to a high degree on the quality of both prognosis and preselection. Does the system of equations contain all the necessary and essential constraints? Have the equations describing the given conditions been determined numerically in the correct manner? Have some already determined activities been considered as variables? Do the most significant and characteristic alternatives figure in the model? All this will depend on the efficiency of prognosis and preselection.

4.2 THE MODEL'S "SUBJECT MATTER"

The model is used to work out recommendations and estimates of the economic activities of production, investment, technical development, product distribution in the productive sphere, and foreign trade. Let us call this the *subject matter* of the model. Some additional subjects, which do *not* fall within that subject matter, should be mentioned.

1. The patterns of public and private *consumption* must be de-

termined from outside for the model. They may be given in several variants, but their determination must take place outside the model.

2. The wage-fund quota must be determined from outside. It will then be allocated to the branches by the model. This, however, is only part of the planning of *income distribution;* the rest must be planned outside the model.

3. The labor quotas must be fixed from outside. These too will be allocated to the branches by the model. Again, this is only part of the work of *manpower* planning – the rest must be worked outside the model.

4. The outputs meant to serve for *stockpiling* are prescribed for the model. The suitable volume of stock must be established outside the model.

5. The model has no *regional* aspects and does not provide recommendations for the territorial allocation of production resources. It does not take into direct consideration such *social* consequences of economic development as urbanization and the changes in social stratification. This will call for computations outside the model and for analysis not only in the economic but also in the sociological and other domains.

6. The model's sensitivity tests and shadow price system provide important bases for *price formation.* The model itself, however, is no price model; it leaves out of consideration essential relationships that must be taken into account in practical price formation.

7. The model does not supply information about the suitable choice of the *economic instruments* necessary for further implementation of the plan.

Thus, *the subject matter of the model embraces only part of the subject matter of medium-term planning.*

The method for the planning of the subjects listed above, as well as for those not mentioned, is highly relevant to the quality of planning in general. It will make a great difference whether it is based on primitive forecasting or on such sophisticated methods as mathematical-statistical analysis, international comparison or on special models of consumption, income distribution, labor-planning, price formation, etc.

4.3 PARALLEL CALCULATIONS AND MUTUAL CONTROL

Sections 1.1 and 1.2 have dealt with the planning functions that *complement* those performed by the model described here. But in addition to complementarity, some degree of parallelism is needed for calculations "competing" with one another.

The model – like all plan computations – *works with definite simplifying assumptions. It may prove useful to carry out other parallel calculations that are free of such simplification.* The control calculations may even employ other simplifications. For example, the model works with continuous variables and must therefore disregard the phenomena of indivisibility: below a certain plant size, establishing a new plant is practically useless. Thus, it is worthwhile to carry out parallel plan calculations that do not disregard this and that are based either on intuitive methods of traditional, nonmathematical planning or on procedures of discrete programming.

The data of the model were based mainly on expert estimates, as described in section 3 of Part I. It is therefore necessary to carry out parallel *calculations* based on other sources of data, especially on full-scale statistical observation and on econometric estimates.

The parallel calculations seek mutual control, the reciprocal disclosure of errors. The results confirmed by both calculation series will provide a firmer basis for decision-making.

5. REQUIREMENTS OF RATIONAL PLANNING AND THE MODEL

In Part I, sufficiently modest limits were set for the scope of action within which the model may perform definite planning functions. Our subsequent analysis will remain within the same limits.

In the analysis, some *requirements* will be set to serve as guiding principles. In my view *the work of planning may be termed mature and rational if it meets these requirements.* Only the *necessary conditions* will be defined; these are, however, not sufficient to determine the complete requirement system of rational planning.

In the discussion the requirement will form an introduction to subsequent explanations and conceptual definitions.

Requirement 1. The plan should contain the basic regulation variables affecting the structure of the economy that are at the command of the "addressees."

The plan offers recommendations to such institutions as the central organs of economic administration, the ministries, the medium-level control agencies, large enterprises, banks, etc. – the "addressees" of the plan. Every addressee has the power to *regulate* certain definite processes and magnitudes affecting the structure of the economy. Regulation means that the trend of the process, the development of the magnitude, depends basically on the addressee's activities. The index that gives the planned and recommended value of the economic process or magnitude is the "regulation variable."[12] All other index numbers of the plan's index system will be called *prognostic* variables.

Regulation variables are within the institutional framework of the Hungarian economy, for example, the allocation of investment proposals for major projects, or the export obligations that can be undertaken under a long-term foreign trade agreement. The growth rate of national income, on the other hand, is a prognostic and not a regulation variable. The government cannot determine the growth rate; it can only take certain steps that will affect its trend in the long run.

In the old (pre-1968) mechanism,[13] traditional planning did more or less meet Requirement 1. *The multi-level programming model of the national economy was – in contrast with a great number of other mathematical planning models – also constructed to meet this requirement. This is one of the main causes of the* *model's highly detailed and disaggregate character.* Otherwise it would not have been possible to build the essential regulation variables of investment and foreign trade into the model.

Many people believed that Requirement 1 had to be met only within the framework of the old mechanism. According to these views, under the new (1968) mechanism, the plan would have to contain only the "basic" and "principal" indicators. Any further breakdown of the plan – characteristic of the overcentralized old system of economic control – would become unnecessary.

I believe these views are erroneous. Reform of the economic mechanism will naturally necessitate reexamination of the plan index system and thus also revision of the regulation variables. Which institutions should be the plan's addressees under the new conditions? What economic processes will the addressees actually be able to control and regulate? How will the economic administration make the lower-level addressees of the plan realize the planned values of the regulation variables addressed to them? These questions require careful analysis. But, although the revision is justified, the requirement itself must be upheld. Any index system that does not meet Requirement 1 cannot be considered a plan and action program, only a prognosis.[14]

In practice, a continued need exists for planning models that contain, in a comparatively detailed breakdown, the major (partly centrally initiated) investment proposals and the main items of long-range international trade agreements. In this connection, too, certain modifications may be necessary (e.g., the breaking down of investments by their financial sources). This will, however, not essentially affect the degree of breakdown in the model and in the index system linked to it.

Requirement 1 represents only a reformulation of the traditional planning practice that gives "priority" treatment to the production of some particularly important commodity group, to the start of some particularly important group of investments, etc. This practice must not be discontinued; that would mean relinquishing control over the structure of the national economy.

Regulation of every detail would be impossible and should not be attempted. Keeping the most important processes under control, however, is both desirable and possible. "Priority treatment" should be based, of course, not on random choice but on selecting the processes that, with their secondary effects, determine the structure of the national economy.

Requirement 2. The plan should be comprehensive and contain the principal aggregate indices of the economy.

The traditional planning methods did more or less satisfy Requirement 2, as did – and more consistently than traditional planning – the input-output tables used for planning purposes, as well as the aggregate programming models based on them.[15] The multi-level programming model of the national economy, on the other hand, failed to meet this requirement in its first experimental application. In further applications it should be possible to remedy these shortcomings. The model's index system must be extended to contain – in addition to (and not instead of) the partial "priority" production, investment and foreign trade estimates – the global figures of production, distribution, investment, and appropriation. It is desirable that – in addition to (and not instead of) physical indicators – the basic estimates of input and output, expressed in terms of value, also appear in the model.

This extension and amplification lead to a series of difficult methodological problems. A connection must be established between the physical indices of priority products and the figures on global values; the nonpriority activities of a residual character must be computed, and so forth. Solution of these methodological problems has already been discussed in planning circles, also independently of the development requirements of economy-wide programming.

Requirement 3. Communication should occur between the aggregate and disaggregate plan indices.

Comparison and analysis of the consequences of alternative economic policies, and the high-level decisions based on them, can take place only on the basis of plan proposals conforming to Requirement 2 – i.e., on aggregate indices. On the other hand, analysis and decision-making will have to be followed, in accordance with Requirement 1, by the concrete definition of the regulation variables. This necessitates *communication* between the aggregate and disaggregate indices.

With traditional planning methods, Requirement 3 is hardly ever satisfied, although attempts are usually made at an approximation, at "breaking down" the economic policy characterized by the aggregate plan figures.

The majority of mathematical planning models would not undertake this task. One of the most important characteristics of the methodology of multi-level planning is that – as has been pointed out in the first part of the paper – in its own sphere of action it fully satisfied Requirement 3. *To every higher-level, aggregate economy-wide program it will be possible to assign a detailed program at any time – in this case, one of about 2,400 variables – i.e., the disaggregate determination of the regulation variables of investment and foreign trade.*

Requirement 4. The system of plan indices should be consistent.

The following discussion will make a strict distinction between the consistency of the *system of plan indices* – the freedom of the system from logical contradictions – and the same property of the *actual plan figures.*[16] The demands made on the former should be made clear by three subrequirements.

Requirement 4.a. The system of plan indices should be logical.

The definitions, classifications, units of measurement, price factors, etc., relating to the system of plan indices should be defined so unequivocally and applied so logically that the operations (addition, multiplication, etc.) performed with the plan indices can be strictly interpreted and the figures to be compared can be truly compared, referring to the same sphere.

Although the requirement may seem trivial, it has been neglected in practical planning. In regard to consistency of the index system, planning is decidedly lagging behind statistics and accounting.[17]

The multi-level programming model of the national economy strictly enforces Requirement 4.a. within its own sphere of action.

Requirement 4.b. The relationship between the various plan indices should be explicable and deducible.

When formulating this requirement, the question of the degree of exactitude in the deductions and explications was left open. All that is required here is the ability to describe the logical process leading from one plan figure to the other or, in general terms, that leading from some *information input* through the plan computation to *the information output.*

Traditional planning can deduce and explain many types of relationships, at least verbally or in the form of simple equations and balances. A great number of other relationships will, however, remain unexplained. They will not be subjected to deduction, not even mentally, or, if so, not described, and their explication not sufficiently controlled. The national economic plan is usually not supplemented by a detailed explication and documentation of the plan computations.

Within its own sphere of action, economy-wide programming carries out a strict deduction of the relationship between the plan indices. *One can always reproduce the way in which the information output (the program) arose from the information input (the coefficients, the constraints, and the objective function).* The application of economy-wide programming may thus indicate significant progress in satisfying Requirement 4.b.

Requirement 4.c. The plan computations should describe as completely as possible the relationships and proportionalities, adherence to which is essential for implementation of the plan.

Requirement 4.b. calls for simple explanation of the plan figures. Requirement 4.c. goes further, calling for the *most extensive and complete description of the relationships.*

Mathematical programming means taking strictly into account all the relationships built into the model, while completely neglecting the relationships that do not figure in it.

With traditional planning, the case is entirely different. The National Planning Office, the ministries, and the various enterprises have several thousand experienced planners on their staffs. Taken together, these planners actually have considerably more relationships in mind than the largest mathematical model – not a few thousand, but tens or even hundreds of thousands. On the other hand – as pointed out in connection with Requirement 4.b. – most of these relationships would not be documented. Thus, it will usually not be possible to ascertain whether or not the relevant relationships had been taken into account.

The tasks to be carried out to satisfy Requirements 4.b. and 4.c. are closely interlinked. The problems lie in several directions. A need exists to expand the sphere of documentation on plan indices, plan relations, and planning computations, also apart from the needs of the mathematical planning projects. The relationships that in many cases existed only in the thoughts of individual planners, never finding verbal expression, or that were, at best, stored among the mass of computational material, should be put in writing and made accessible to other planners as well.[18]

Further, the system of constraints in mathematical planning should be made more complete. Besides Requirement 1 (the establishing of regulation variables), Requirement 4.c., *the control of the plan's workability, warrants great detail and a comparatively high degree of disaggregation in the mathematical planning model.* The aggregate models may more easily shift toward unworkable sectoral patterns than the multi-level model, which takes into account conditions of foreign trade, technology, natural endowments, etc.

Of course, the constraint system of a mathematical model can never be "complete." The limitations are partly of a computation-technical character. The programs yielded by the mathematical methods should therefore always be checked by practical experts. Tests should be made of whether the program that is feasible from the point of view of the model is also consistent with certain proportionalities, conditions, and relationships not covered by the model but sensed by the practical planner.

Requirement 5. The plan should be workable and ensure an equilibrium.

Requirement 4 called for consistency and freedom from logical contradictions in the system of plan indices. The present requirement calls for freedom from contradictions in the plan targets as well.

The concept of equilibrium should be interpreted as follows.

No deficiency of products and resources covered in the system of plan indices should exist. A surplus may occur, but the planners should know its extent and take into consideration such consequences as the accumulation of stocks, the unemployment of certain groups, etc.

Because of unforeseen circumstances, the actual extent of the resources or inputs may in reality differ from that envisaged in the plan. Also, Requirement 5 does not unrealistically demand complete realization of the plan. Rather, it does call for the fact that, in the course of planning, utilizing all given and available information, every factor should be carefully taken into account that may impede the plan's implementation. *A relative equilibrium* of all relationships in the plan should be attained *as far as the supply of information permits*, by minimizing potential equilibrium disturbances.

The multi-level programming model of the national economy, with its highly disaggregate structure embracing some 2,000 relationships, strictly enforces Requirement 5 within its own sphere of action. At the same time, in the first concrete model of the experimental computations, several equilibrium and proportionality relationships were neglected, which, if considered, would have required summary in value terms. As noted, with the new models to be developed later on, it should be possible to eliminate this deficiency.

With Requirement 5, economy-wide programming has a marked advantage over traditional planning methods. No intuitive method of plan coordination, no repeated verbal or written discussion, can compete with the strict internal logic of a mathematical equation system.

Requirement 6. The plan should be at least approximately efficient.

A plan will be called efficient if it is workable, ensures an equilibrium (i.e., satisfies Requirement 5), and cannot be confronted with another equally workable plan that is not less advantageous from any point of view and more advantageous from at least one. For example, both Plan 1 and Plan 2 may be efficient if one envisages higher private consumption and a less favorable balance of payments, and the other envisages lower private consumption and a more favorable balance of payments. On the other hand, Plan 3 will be inefficient if it is less advantageous than Plan 1 in terms of private consumption and balance of payments. In that case, Plan 1 will *dominate* Plan 3.

Traditional planning cannot even approximately satisfy Requirement 6, if only because it cannot fulfill either Requirement 4 (particularly not 4.b.) or Requirement 5. No question of efficiency exists if a plan's estimates are not explained and deduced in every detail, its index system is not consistent, and its equilibrium cannot be demonstrated. Under these circumstances, whether or not it dominates the other plans which it may confront cannot be known.

Multi-level economy-wide programming can satisfy Requirement 6. True, it will only approximate full efficiency. First, as was noted under Requirement 5, it does not ensure complete workability. Second, it has to use an approximate method instead of an exact procedure. It has nevertheless succeeded in working out plan proposals that are significantly more advantageous from several aspects (representing considerable savings) than plans based on traditional methods.

Table 2 presents the results of five programs worked out on the basis of the economy-wide computation series. None of the five programs dominates any of the others, and all are approximately efficient.

Table 2

Savings and Surplus Returns

Programming problem	Unit	Saving (surplus return vs. official program)
Minimizing investment inputs	Billion forints	15.5
Minimizing live labor inputs	Thousands of manpower	253
Optimizing the net balance of socialist foreign trade	Million rubles	126.8
Optimizing the net balance of capitalist foreign trade	Million dollars	122.2
Maximizing surplus consumption	Billion forints	7.53

With further development of the economy-wide programming model, improvement in computing techniques, and application of exact procedures, progress can be made toward replacement of the present approximate computations by methods satisfying Requirement 6.

Requirement 7. Planning organs should submit to the political decision-making bodies the political plan variants made ready for decision. The variants should provide choices conforming with timely political decisions and present the consequences of the alternative choices.

Political plan variants are the plan variables that differ from each other in such essential political consequences as the planned standard of living, the rate of increase in the production fund, the orientation of foreign trade and international credit policies, etc.

A plan variant may be termed *ready for decision* if it satisfies the six requirements noted above. This will make it possible for the supreme decision-making bodies to survey the basic variants, each of which is in itself consistent, workable, and nearly efficient, and can be "translated" into regulation variables, i.e., into indices whose regulation is actually in the hands of the state.

Traditional planning cannot prepare simultaneous political plan variants. Multi-level economy-wide programming, on the other hand, can do so within its own sphere of action. This is one of its main purposes. A great number of economy-wide plan variants have already been prepared in higher-level computations. The methodology of these computations makes it possible for every variant to satisfy in itself the constraint system described in the model and to be approximately efficient. Moreover, for every variant characterized by its principal indicators, it will be possible to give a detailed program broken down by regulation variables.

Requirement 8. The period of regulation should be as short as possible, the time horizon as long as possible.

To formulate this requirement, two new concepts have been introduced: first, the *period of regulation* – the period for which the *planned* value of the regulation variables basically affecting the structure of the economy is *fixed* in advance. The definition stresses two words: "fixed" – the planned value of the regulation variables will not change in the regulation period; and "planned" – fixing refers only to the suggested value of the regulation variable, without excluding the possibility that the actual magnitude will differ from the planned one.

The second important concept is the *time horizon.* This is the period for which the consequences of the planned value of the regulation variables are estimated and forecast.

The logical point of departure of Requirement 8 is that the value of the structural regulation variables should be determined in the most reliable manner. This requires, on the one hand, basing them on the most recent information – for example, the investment decision should be as close as possible to the start of the investment project (a short period of regulation) – and, on the other hand, considering the lasting consequences of their interrelations most carefully (a long time horizon).

Ideally, the period of regulation will be one year and the time horizon will be between fifteen and twenty years or, in certain relations, infinite. For the time being, as a first step in the development of planning methodology, less favorable solutions may also be termed satisfactory. For example, the regulation period may be between two and three years – the working out of a five-year plan, with a revision of the plan around the middle of the period. The time horizon may be twelve to fifteen years – fifteen when the original five-year plan is worked out, and the remaining twelve at the time of its revision.

Traditional, nonmathematical planning has made a start in this direction. Revision began in the middle of the present five-year plan. Preparations were started for the so-called long-term plan that could give an adequate time horizon to the new five-year plan covering 1971-1975.

Even in its first formulation, the multi-level programming model of the national economy can satisfy the first half of Requirement 8. (This question will arise in Requirement 9.) The first experimental computation, however, did not have a lengthy time horizon reaching beyond the regulation period. The model must be developed with a time horizon reaching beyond the five-year term – perhaps one of fifteen years. The model for the second and third periods should not need the same detailed breakdown (2,400 variables) as the first, because the value of the regulation variables has to be computed only for the first period of regulation.

The methodological problems of the model's dynamization, of its transformation into a multi-periodic model, cannot be dealt with here.

Requirement 9. Planning must be continuous. The methodology of planning should permit the continuous processing of fresh information and a speedy revision of the plan computations.

Requirement 9 follows from Requirements 7 and 8; it is that of continuous planning. Reality is continuously changing; continuous change occurs in available technical, economic, and political information, in procedures and forecasts, in instructions from political bodies, and so forth.

A planning methodology must consider the continuous modifications in the information material as natural. The structure of the system of plan indices should be comparatively stable. Classifications and nomenclatures should be as permanent as possible; the definitions of the indices should possibly not change but remain comparable; no major changes should occur in the system of data supply, in the arrangement of forms, etc. The numerical contents of the system of plan indices should also be as up-to-date as possible. Continuous revision of the already completed plan computations should be ensured. Information processing and the computation of secondary effects should be rapid; the procedures of plan coordination, revision, and variant computation should be easily and speedily repeated.

Traditional planning is unable to cope with Requirement 9, although this would be needed over and over again. Such complaints as "Everything has been upset again..." or "The data supplied by this or that institution, person, agency, etc., are different from last time...," are frequently heard. Elaboration of a five-year plan generally takes several years, and although simultaneous variants are not worked out, the plan will, in the course of time and planning, repeatedly undergo modifications; but it will usually not be possible to carry out a consistent correction of the earlier plan proposals, to work out systematically all secondary consequences of partial changes. This task is practically insoluble by "handicraft" methods.

The methodology of economy-wide programming, on the other hand, allows for continuous planning. A model is worked out, together with the index system belonging to it. All data pertaining to the model are stored on punched cards and tapes, and on magnetic tapes. The individual partial computations can be performed with great rapidity. A change in any data or data group will simply be made at the corresponding place of storage (the old punched card replaced by a new one, etc.), and the consequences of the modification can be assessed immediately.

Herein lies the main strength of mathematical planning – the ability to recompute a plan with new information material. This is only a methodological possibility, however; utilization will depend on personnel, organizational, and technical computing conditions (see below).

6. PRACTICAL CONDITIONS OF SYSTEMATIC APPLICATION

This paper is not meant to foster illusions. Multi-level economy-wide programming cannot pretend to provide a solution for every planning problem. Certainly, however, as convincingly proven by the first experimental computations, the application of the model described here would significantly advance the development of Hungarian economic planning.

The first computation took considerably more time than originally expected – four years, not counting periods of theoretical preparation and detailed evaluation of the completed computations.[19] Practical application of the method will be possible only if the next computation can be carried out within a much shorter period. The results concerning the economy as a whole must be available before the beginning of the regulation period, when the plan decisions are actually made.

Let us now survey the personnel, organizational, and technical computing problems of practical utilization and of speeding up the computations.

6.1 THE MACHINERY

At the height of the first experimental computation, the project engaged six to ten full-time research economists, four to five full-time computing-technical mathematicians, twelve to fifteen part-time economists, four to five laboratory assistants, joined by 150 to 200 occasional participants, such as data suppliers, advisers, and those carrying out partial tasks. This large panel was composed of members belonging to about thirty different institutions – scientific institutes, computing centers, and high- and medium-level economic institutions and enterprises.

The collective consisted entirely of *volunteers*. Each was invited and first asked whether he or she would be inclined to join. Only when this was cleared would the invitation be "legalized" by asking the permission of the superiors of the person concerned the official authorization of the institution in question.

With this volunteer team, a high degree of centralization could be achieved. Methodological unity was ensured more strictly than in the case of the traditional index system which, although enforced by state discipline, is less logically consistent – all this in spite of the fact that those directing the research project had absolutely no "authority" to demand enforcement of the methodology.

The project had a strange mixture of team-forming motives. It contained the elements of a *"movement"* – the optimist's belief in a fine idea, in rational mathematical planning, expressed in enthusiasm, unselfishness, and voluntary discipline. It also contained the elements of the *scientific "team"* – the joy of joint intellectual excitement, of joint discovery and thinking, as a driving force. Undeniably, the elements of "work on the side," so characteristic of present-day conditions, were present; many participants earned extra money from the project.

Clearly, it would be impossible to repeat the project in this form. Enthusiasm will not last forever; toward the end of the work, even that of the most unselfish participants showed signs of abating. Scientific interest is also bound to decline when the excitement of discovery is over and the problems are confined to practical application of a novel method. Finally, as regards "work on the side," it certainly cannot provide a lasting basis for the systematic application of a specially important method.

Economy-wide programming must abandon the working methods characteristic of a "movement " a "scientific team," and a "side activity," and adopt those of official work, preserving, of course, to the highest possible degree, enthusiasm and intellectual standards. Economy-wide planning should change from "nonprofessional" to "professional" activity.

Voluntary discipline is very preferable to enforced discipline, as long as one displays a will to work in a disciplined way. If will is lacking, however, one cannot interfere. Delays are frequently caused by slackness and lack of discipline, or by the fact that participants were assigned another task by their own institution or superiors. It is a characteristic fact that the *first* sector computations were finished in 1964, which proves that it is possible to complete a sector model within three-quarters of a year. In some sectors, however, the model was not ready until early 1967, two and a half years later.

If the economic administration intends to complete future computations of this character in less time, it will then be necessary to create an apparatus of suitable dimensions, whose main purpose is to carry out economy-wide programming and observe the time limits set for its tasks.[20] This apparatus should be served by experts well versed in mathematical planning methods and operate within the central and medium-level planning organizations themselves.

6.2 STANDARD OF KNOWLEDGE OF THE PRACTICAL PLANNERS

At the start of the research project, the training of programming experts had been set as one of the primary objectives. The results achieved may be termed satisfactory. In the beginning, there were probably three or four participants who had previously engaged in practical mathematical programming, at least on the sector level.

The rest had some knowledge of the method from literature, or none at all. By the time the project ended, at least 40 to 60 research workers were thoroughly trained in the methodology of economy-wide programming, not only theoretically but on the basis of practical experience as well.

In the knowledge of mathematical planning, as in the learning of languages, two degrees exist: active and passive knowledge. The above-mentioned 40 to 60 participants in the programming project have attained active knowledge and should be able to construct a model and to carry out computations with it. The greatest possible number of practical planners, however, must acquire at least a passive knowledge of the method. As has been pointed out in detail, the model is closely linked with its environment, with planning work as a whole. The practical planners supply data and assist in the construction of the model – in the selection of the variables, as well as in working out the system of constraints – and in the practical evaluation of the results obtained. They *put the questions* to the model and process the answers received. All this requires at least an elementary knowledge of the language of the model. In other words, *every practical planner should know the conceptual system and general logic of mathematical planning,* even if he has not mastered the technique in all its details.

Unfortunately, little has been achieved in this field. Experience shows that the ideas of mathematical planning have scarcely been absorbed by the many thousands of central, medium-level, and enterprise planners. The material and the literature on the subject have not been extensively studied. Analysis of the reasons would lead us too far; let a statement of the fact suffice.

A radical change in the training and retraining of practical planners is an essential precondition of the systematic practical utilization of multi-level economy-wide programming and of mathematical planning in general. Up-to-date planning methods should be taught extensively and systematically.

6.3 THE SUPPLY OF DATA

The economy-wide programming model has been constructed to conform as much as possible to the index system of traditional planning. This was done to utilize to the greatest possible extent the information material of the Planning Office and the ministries.

In retrospect, this procedure was correct. It would have been almost impossible to procure all the model data unaided. Even so, the task proved more difficult than expected. Considerably more data than expected had to be collected from sources outside the official documentation or worked out specially for the purpose. This was one of the main reasons for the protracted work.

What exactly were the causes of the difficulties in data supply, and how can they be eliminated in the future?

One problem has already been pointed out in connection with consistency – Requirement 4 of rational planning. The whole of traditional indices does not form a coherent system free of logical contradictions.

Systematic application and further development of economy-wide programming are inseparably connected with development of the plan index system as a whole. Mathematical modeling, which, with its requirement of consistent data supply, is in great need of general regulation, should become one of the beneficiaries of the result.

Even independently of the problems of multi-level mathematical programming, the general development of the plan index system is under consideration. *The system of indices employed up to the present needs thorough revision.* Stable definitions and classifications should be worked out, and uniform forms and data documentation prescribed – all done so as to *coordinate completely the data requirements of statistics, nonmathematical planning, and the essential plan models.* This would also permit the mechanical data processing of the plan information material.

A further problem: traditional planning collects few data that can be used for the computation of *variants*, whether lower-level (investment projects based on alternative technologies) or higher-level (alternative economy-wide plans).

In connection with the data, we should mention *information responsibility*, as in this recurring experience:

A figure – say, an export constraint – was supplied by a member of the official planning staff. It was inserted in the model, the computation was run through, the program exhausted the constraint, and the export figure reached the permitted maximum. Then the planner who had supplied the figure quickly withdrew it, saying that it was in fact impossible to reach. One of the most important characteristics of mathematical planning is that the model, the computer, cannot differentiate between seriously validated figures and those thrown in irresponsibly: it will deal with every figure identically. Those in charge of official planning work have developed a certain instinct for dealing with the figures submitted – to increase or decrease them according to what they refer and by whom they were submitted. The computer has no such instinct. This situation proved rather unusual to those supplying the data, and a great number of computations had to be repeated.

In the long run, economy-wide programming will require neither more nor other data than traditional planning in the same sphere of action. After appropriate coordination, *all information required for the quantification of the multi-level programming model should be collected through the channels of official plan data supply.*

6.4 COMPUTING TECHNIQUES

The major reasons for the protraction of the first experimental computations were severe computing technical difficulties. These difficulties were not of the same character as those usually encountered by such pioneers as the first steamboatmen or flyers. As a matter of fact, it should have been possible to provide for the necessary computing technical conditions at present-day standards from the country's given resources (and even from the foreign exchange spent on computers throughout the country in the past few years) and with the existing Hungarian staff of mathematicians and computer technicians.

In the project's four to five years, six different computing centers had to be used. Because we correctly sought to employ the largest computer available in the country, the type had to be changed three times. Each time the laborious procedure of working out and running in the computer programs had to be started again. All this was aggravated by the fact that economic planning in Hungary has no computing technical base of its own; no computer in the country primarily serves the purposes of planning. The economy-wide programming project had to be carried out on computers belonging to various institutions that were not always cooperative. This enormously delayed the working out of new computer programs and the computations based on them.

An essential precondition of the operative application of economy-wide programming (and of mathematical planning in general) is that the planning apparatus should be served by an adequate computing technical base of its own - a *large* computer technically suited for rapid and reliable solution of extensive programming problems with high data requirements, together with an appropriate staff of mathematicians and attendants. These are quite trivial conditions that should not involve any objective difficulty.

6.5 MULTI-LEVEL PROGRAMMING AND THE GENERAL DEVELOPMENT OF PLANNING

Sections 3.1.-3.4. gave a survey of the factors that lengthened the execution of the first experimental computation, as well as of the conditions essential for the method's further (and speedier) application. Now, will these conditions be realized?

Certain economic models may be worked out in the quiet rooms of a research institute, quantified on the basis of printed statistics, and published together with their results in periodicals without research workers ever being in contact with practitioners. The multi-level planning model is not one of these. It is linked by a thousand threads to the living machinery of planning, from the data requirements and the practical advice needed for its concrete construction to the utilization of the results. *The model will either succeed in fitting organically into the living work of planning or be eliminated.*

The destiny of the model was previously dependent on research workers who initiated the work and carried out the first experimental computations. It needed only benevolent backing from practice – and this was usually granted.[21] Now, however, a new phase has been reached: application. The destinies of multi-level economy-wide programming have passed to the hands of practical planners, first of all to those responsible for the control of planning work. Mathematical economists and research workers will continue to play an important part; they must help in training the staff and in developing the methodology. But it will obviously be beyond their power to organize the official machinery required for practical application and for large-scale retraining of practical planners, to transform fundamentally the system of plan indices, and to build the computing technical base of planning. All this should be left to economic administration, to those in control of the planning machinery.

This paper has dealt throughout with the conditions of application of multi-level economy-wide programming. The problems are closely interrelated, however, with those of the general development of planning. The present model is not alone in requiring mathematical planning experts in the official planning machinery, the up-to-date retraining of practical planners, the reorganization of the supply of planning data, and the establishment of a computing technical base for planning. All this has by now become timely. Practical application of multi-level economy-wide programming will be only a *function* – and, at the same time a clear *measure* – of the further progress toward rendering planning more rational and raising its intellectual standard.

FOOTNOTES

1. The basic ideas of combining the various sectoral planning models were outlined in the author's paper "The Linking of Central and Sectoral Programming Projects" (Budapest, 1961, Computing Center of the Hungarian Academy of Sciences, in manuscript). In 1962, another paper on the subject was published in co-authorship with Th. Lipták under the title "Two-Level Planning" (Budapest, Computing Center of the Hungarian Academy of Sciences, mimeographed; published in *Econometrica*, 1965, No. 33, pp. 141-169). Preparations for practical calculations started in December 1963.
2. The project was directed by a central team headed by the author and composed of members of the Computing Center of the Hungarian Academy of Sciences and of the Institute for Economic Planning of the National Planning Office.
3. Some sectors were entirely left out only because of practical organizational reasons, namely, the failure to form research teams to work out the corresponding part of the model. All the branches of the economy are not covered in these first experimental computations; the method could be tested without that. Even so, the model ultimately became larger and more comprehensive than intended. As a matter of fact, it was originally planned to cover 10 to 20 sectors only.
4. The 74 sectors taken into account in the comparison do not include the three private sectors, the three residual sectors with a "sundry" character, and the handicrafts sector.
5. The term "traditional planning" describes the non-mathematical planning methods used in the Hungarian planned economy for the last twenty years.
6. The schema contains only the coefficients of the activity variables; the unit vectors belonging to the auxiliary variables and containing the coefficients of the latter form a further block of the programming problem's matrix of coefficients, which is not represented here.
7. With the present world level of computing techniques, this should not be impossible. Linear programming problems with several thousand constraints have already been solved abroad with direct methods, without decomposition.
8. G. B. Dantzig and P. Wolfe, "The Decomposition Algorithms for Linear Programs," *Econometrica*, 1961, No. 29, pp 767-778.
9. J. Kornai and Th. Lipták, "Two-Level Planning," *Econometrica*, 1965, No. 33 pp. 141-169.
10. The method has been worked out by the author. For a detailed Hungarian description see "Közelítő eljárás lineáris programozási feladatok dekompozíciós számítására" [An Approximation Method for the Decomposition of Linear Programming Problems], *Szigma*, 1969, No. 1, pp. 26-46.
11. At the beginning of the research work, it was noted that the computations "...should be considered a scientific experiment whose main importance lies in testing the new planning method.... All this is emphasized here exclusively to make it clear from the beginning that the fact that the computations may provide a basis for practical planning decisions was considered of secondary and additional importance only.... It is deemed most important not to raise ex