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Investment, Efficiency, and Shortage: A Macro-Growth Model

I. Introduction

Our choice of a theme for this article was inspired by the present economic difficulties of the Eastern European socialist countries. We would like to describe the connections between the following processes: (i) The intensity of shortage in some of the above countries is not decreasing significantly, and in a few of them has markedly increased. (ii) The efficiency of both production and investment in several countries has sharply deteriorated. (iii) The growth of investment, production, and consumption has slowed down a lot, and in some cases ceased altogether. (iv) Some socialist countries strive to maintain the rate of growth of capacity by continuously raising the investment ratio—whether this is at the cost of restricting consumption or of intensifying shortage.

The phenomena just listed are clearly interconnected. For example, declining investment efficiency combined with a constant investment ratio gives rise to slower growth of capacity, and this can only partly be counteracted by raising the investment ratio;

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moreover, the worsening shortage caused by overinvestment causes further falls in investment efficiency.

In this study we should like to bring together two areas of the economic literature. The first area is *growth theory*, which traditionally concentrates on examining the relationships between the growth rate, the investment ratio, and investment efficiency.¹ It can be shown (though we do not do so in this article because of space constraints) that some of the classical results of growth theory can also be established for our own model, or at least for a suitable specification of our model. The second area is the *descriptive theory of the socialist economy*, more exactly, that variant of the theory developed in Hungary and summarized in the work of Kornia (*The Economics of Shortage*, 1980) [12]. In the framework of this type of theory we have already done some work on modeling the macrodynamic behavior of the socialist economy [13, 15].² The present paper follows on directly from this work; for a fuller review of the theoretical background we refer readers to our earlier publications. However, this paper differs in some respects from these earlier works. First, we simplify the earlier model as much as possible: we neglect lags and treat the consumption ratio as exogenous. Second, however, we are able to make some advances. By freeing the previous model from its Neumann-type assumptions, we succeeded in formulating a model in which the growth rates of production, investment, and consumption could be different not only in the short run but also in the longer term. Instead of the rather artificial additive shortage index used before, we now work with a multiplicative index that is more easily understood. As a result of these changes, the new model is analytically more tractable than the old one.

This article is confined to the level of pure theory. In particular, we do not investigate the changes that might be required in the model to facilitate econometric analysis.

In what follows, section 2 introduces the model and the basic assumptions. Then section 3 analyzes the stability and viability of the stationary path. Section 4 analyzes those growth paths along

which worsening efficiency of production and investment slows down the growth of capacity. Mathematical proofs of the formal results are given in the Appendix.

2. The model

2.1. Basic assumptions

Let us begin by stating the basic assumptions; we shall introduce additional, special assumptions while describing and analyzing the model.

1. There is a single, homogeneous product in the macro-model.
2. Prices are fixed and aggregation is always done at these fixed prices.
3. There is no explicit role for money in the model.
4. The consumption ratio is exogenously given.
5. Capacity is the sole factor of production, and it is determined by investment. The labor force plays no part in the analysis and technical progress is not treated explicitly.
6. All output is storable.
7. There are no time lags in either real or control processes.
8. The economy is closed; there is no foreign trade.
9. Time is a continuous variable and the parameters of the model are continuously differentiable functions.

The above assumptions—especially numbers 4 and 8—make it possible to demonstrate some relatively simple and comprehensive relationships. Consumption and foreign trade can also be treated endogenously, and we have actually done this in some of our other analysis. Their introduction would, however, be superfluous in this initial exposition. Aside from this, our procedure is also desirable because it results in a model which directs our attention to the investment process, and this is where we shall find the kernel of an explanation for the regularities revealed in the operation and growth of the socialist economy.

2.2. Variables

[A list of variables follows.]³ For the sake of brevity we omit the argument, t (time).

V = *stocks*. In distinction from our earlier work, only producers' output stocks are treated as a variable; we disregard the fact that producers' and consumers' input stocks should be treated as separate variables.

X = (gross) *production*.

K = *capacity*. Maximum output "theoretically" attainable with fixed capital accumulated earlier and the available input stocks; however, actual production does not reach this level even at cyclical peaks.

H = *consumption*.

I = *investment*. We regard as investment all accumulation leading to a net increase in fixed capital or producers' input stocks. These outlays result in an increase in capacity without any lag.

B = *increase in capacity*.

A = *productive consumption*. This includes replacement investment.

The seven variables listed so far are the usual cardinal indicators which are measured in units of the "homogeneous macro-output." The following, ordinal indicator is not of this type.

Z = *degree of shortage*. This is larger, the greater the shortage; we come back to its explanation later on.

2.3. The equations of the model

Balance equations. By definition the following balance equations—formulated as differential equations—must be satisfied:

Stock change:

$$\dot{V} = X - A - I \quad (1)$$

Capacity change:

$$\dot{K} = B \quad (2)$$

Ratios of variables. Certain ratios of the variables play an important role in our model. Let us set out their names and formulas:

Rate of turnover of stocks:

$$v = X/V \quad (3)$$

Capacity utilization:

$$k = X/K \quad (4)$$

Cost coefficient:

$$a = A/X \quad (5)$$

Investment efficiency:

$$b = B/I \quad (6)$$

Investment ratio:

$$i = I/X \quad (7)$$

Consumption ratio:

$$h = H/X \quad (8)$$

In the traditional *linear* growth models it is quite usual to assume that these ratios are *constants* determined outside the model (exogenously). In this paper the relevant ratios vary with time and certain relationships obtain between them. To describe

these, however, we must first introduce the equation determining the degree of shortage.

The degree of shortage. In our use of the word "shortage," we refer to an extensive range of phenomena, combining millions of elementary micro-events: purchasers are unable to satisfy part of their original, concrete demand; they try searching for the desired good or they accept forced substitution, or they postpone their shopping until later. We represent the frequency and severity of these elementary events—for purposes of macroeconomic analysis—by means of a synthetic shortage index.⁴ The next equation tells us *what determines Z*, the degree of shortage, while the subsequent three equations will clarify what *effect* shortage has on the other variables in the model.

For a complete explanation of shortage in a chronic shortage economy we should require a causal chain composed of many links [12]. In this simple growth model we shall only be concerned with the final links of the causal chain (and only with part of these). Let us suppose that shortage is more intense the higher the rate of turnover of stocks and the more fully capacity is used, that is, the more frequently production runs up against concrete micro-capacity constraints caused by machinery or supply bottlenecks. To show this relationship it would be sufficient to propose a general function $Z(k, v)$, increasing in both arguments. We found it more useful, however, to replace the general function $Z(k, v)$ with the following special form:

$$Z = \left[1 - \left(\frac{k}{k_m} \right)^\sigma \right] \left[1 - \left(\frac{v}{v_m} \right)^\tau \right]^{-1}, \text{ where } \frac{k}{k_m} \leq 1, \quad (9)$$

where k_m and v_m are the *maximum* permitted values of capacity utilization and the rate of turnover of stocks, respectively; σ and τ are positive constants. The minimal value of Z is 1, which occurs when $k = v = 0$.⁵ The maximum value of Z is ∞ , which occurs when $k = k_m$ and $v = v_m$. The larger the value of σ , the more sensitively the degree of shortage depends on capacity utilization and the less it depends on the rate of turnover of stocks. The

larger the value of σ , the faster the degree of shortage increases as capacity utilization increases, while as τ increases, the degree of shortage is more sensitive to increases in the rate of turnover of stocks.

We define the *viability* of the system as follows:

$$0 < v < v_m \text{ and } 0 < k < k_m.$$

Remarks. Two considerations led us to the choice of the particular functional specification (9). First, it is quite a tractable form. Second, the "Cobb-Douglas" form is familiar and well known to economists. Formula (9) adequately reflects the causal relationships in the particular situation where we find both a high rate of turnover of output stocks *and* high capacity utilization often running up against bottlenecks (later on, as a result of taut output plans and "shock work"). On the other hand, the formula does have an important weakness. In a situation of chronic shortage, it usually happens that just because some resource runs up against a tight bottleneck, the utilization of some other, complementary resource is incomplete, and for this reason the overall average utilization of resources is likely to be low. This phenomenon can only be represented satisfactorily in a disaggregated model. The high degree of capacity utilization in our aggregated, deterministic model actually expresses that stochastic micro-level phenomenon whereby the occurrence of bottlenecks in production is very frequent, and bottlenecks have wide-ranging effects. Similarly, we excluded from our analysis any structural aspects of the macro-variables; for instance, a given rate of turnover of stocks can be associated with completely different shortage situations.

The real effects of shortage. While in the above we explained shortage in terms of taut utilization of resources, we are now concerned with effects in the opposite direction. We make increased shortage responsible for the poor efficiency of resource utilization.

Let us imagine first of all a *system without shortage*, where the

material cost coefficient and the investment efficiency, both generally varying over time, would take the values $\tilde{a}$ and $\tilde{b}(i)$, respectively. For the moment we are not concerned about their time variation and merely note that investment efficiency is a decreasing function of the investment ratio, and we regard the parameters $\tilde{a}$ and $\tilde{b}(i)$ as exogenous functions of time.

In our model, however, it is the *actual* cost coefficient and investment efficiency that we need. Let us suppose that the cost coefficient is an increasing, and the investment efficiency a decreasing, function of the degree of shortage. In reality, shortage leads to waste of materials and to a deterioration of quality due to forced substitution. Overstrained investment activity brings about frequent stoppages in implementation and delays in bringing new capacity into production, while also giving rise to cost overruns and a shortfall in relation to the planned level of efficiency. In what follows, we regard $\tilde{a}$ as the *minimum* cost coefficient and $\tilde{b}$ as the *maximum* investment efficiency. We represent the efficiency-reducing effect of shortage by means of a *constant elasticity* function. For each 1% increase in shortage, the cost coefficient increases by $\alpha\%$ and investment efficiency declines by $\beta\%$. In symbols, we have:

$$a = \tilde{a}Z^\alpha \quad (10)$$

and

$$b = \tilde{b}(i)Z^{-\beta} \quad (11)$$

Let us make the further special assumption that the elasticity of the maximum investment efficiency with respect to the investment ratio is also constant:

$$\tilde{b}(i) = \tilde{b}i^{-\omega} \quad (12)$$

where $\omega < 1$ and $\tilde{b}$ is an exogenous function of time.

Remark. The economic conception lying behind functions (11) and (12) is somewhat related to an observation of Branko Horvat [9, 10]: namely, that overstraining investment reduces efficiency.

A similar relationship can be shown in neoclassical growth models⁶ assuming a decreasing marginal efficiency of investment. The difference—aside from purely formal differences—is essentially that we bring investment efficiency explicitly into the relationship, in the first place through the macroeconomic situation, through “utilization” and “tautness.” *Apart from this* our model *also* allows for the possibility that investment efficiency may be a function of the investment ratio.

An additional new feature can be seen in function (10). Almost all growth models treat the current input cost function as exogenous. Here, in contrast, it depends endogenously on the macroeconomic situation, on the degree of shortage.

The regulatory effect of shortage. So far we have been concerned with the real effects of shortage, but now we turn to its control, or regulatory, effects.

We explain the development of the *investment ratio* in terms of two factors. We start with a maximum investment ratio, $\tilde{i}$, exogenously given and varying with time, a ratio that would occur in the economy if there were no shortage. In the real economy, however, decision makers do experience shortage and, albeit unwillingly, they are forced to reduce the investment ratio. (Note that in our model $\tilde{i}$ is not the planned investment ratio, which could by chance be overfulfilled. In our model, $\tilde{i}$ is the upper limit of the investment ratio, from which the real economy must always deviate in a downward direction.) Here too we represent the actual investment ratio by means of a power function. For each 1% increase in shortage, the investment ratio falls by $\iota\%$:

$$i = \tilde{i}Z^\iota \quad (13)$$

Turning to the description of capacity utilization, we must emphasize that it would now be inconvenient to use the shortage indicator, Z , as a control signal, since Z also depends on k and we should therefore get an implicit equation. For this reason we confine ourselves to taking into account the following relationship: the current production is controlled on the basis of stock

signals. If the rate of turnover of stocks increases relative to its *norm* (and this in itself signifies an increase in the intensity of shortage), production and hence also capacity utilization must increase relative to their respective norms, and *vice versa*.⁷

Let k^* and v^* represent the *norms*, which are invariant over time, of capacity utilization and the rate of turnover of stocks, respectively. Now let us assume that actual capacity utilization is proportional to its own norm and that it rises by $\alpha\%$ for each 1% increase in the stock turnover rate relative to its (i.e., the stock) norm:

$$k = k^* \left(\frac{v}{v^*} \right)^\alpha, \text{ where } \alpha < 1. \quad (14)$$

Remark. It is essential to exclude the cases where $\alpha = 1$, because when $\alpha > 1$ the production rule $X = X(K, V)$ expressed in the original variables would make higher output correspond to higher stocks; when $\alpha = 1$, however, the production rule is not defined.⁸

Finally, let us assume that the actual value of the *consumption ratio* agrees with the value it would take in a system without shortage, which is later taken to be an exogenously given function:

$$h = \bar{h}. \quad (15)$$

Summarizing the equations of the model, we have:

$$a = \bar{a}Z^\alpha, \quad b = \bar{b}Z^{-\beta+\omega}\bar{v}^{-\omega}, \quad i = \bar{i}Z^{-\epsilon}, \quad k = k^*(v/v^*)^\alpha, \quad h = \bar{h}. \quad (16)$$

where

$$Z(v) = \left[1 - \left(\frac{k^*v^\alpha}{k_m v^{*\alpha}} \right)^r \right] \left[1 - \left(\frac{v}{v_m} \right)^r \right], \quad (17)$$

2.4. The fundamental equation

We can simplify the analysis of the model by selecting one of the variables to be the *key variable* and formulating a differential equation in terms of this variable, which we call the *fundamental equation*. As we shall see, it can be verified that if we determine the development path of the key variable, then—by using the original system of equations—we can also uniquely determine the paths of all the other variables.

For reasons of convenience we chose the rate of turnover of stocks as the key variable (i.e., v) (though in fact we could have chosen other variables in the model for just the same reason).

Above all we need to determine the *rate* of increase of capacity. Similarly to the approach of Harrod [6] and Domar [3], it satisfies the following identity: the rate of increase of capacity = capacity utilization multiplied by investment efficiency multiplied by the investment ratio. We denote the rate of growth of capacity in systems with and without shortage by r and $\bar{r}$, respectively, and represent the above identity in the mathematical form:

$$\bar{r} = k^*\bar{b}(\bar{v})\bar{i} \text{ and } r = kb(v)i. \quad (18)$$

The connection between the two growth rates is as follows.

$$r = \bar{r}Z^{-\beta-\omega} \left(\frac{v}{v^*} \right)^\alpha, \quad \bar{r} = k^*\bar{b}\bar{v}^\theta, \quad \theta = 1 - \omega. \quad (19)$$

Now we arrive at the fundamental equation:

$$\dot{v} = \frac{v^2}{1 - \alpha} [\bar{h} - g(v)], \quad (20)$$

where

$$g(v) = 1 - \bar{a}Z(v)^\alpha - \bar{i}Z(v)^{-\epsilon} - \bar{r}Z(v)^{-\beta-\omega}v^{\theta-1}(v^*)^\alpha. \quad (21)$$

Lemma 1. The differential equation (20) has a unique solution on the interval $(0, v_m)$.

Lemma 2. The original model has a unique solution, which can be determined from the solution of the fundamental equation.

Proofs of both lemmas are given in the Appendix.

3. Stability and viability

In this section we investigate the fundamental equation. The results obtained in this way can easily be transferred to the original model.

In dynamic analysis the notion of a stationary point plays a central role; if our dynamic system starts from such a point then it would remain there. In symbols:

$$\text{If } v_0 = \bar{v}, \text{ then } v(t) = \bar{v} \text{ for all } t. \quad (22)$$

From equation (20) stationary point(s) can be determined as the root(s) of the equation:

$$g(v) = \tilde{h} \quad (23)$$

which also implies that $\dot{v} = 0$.

Remark. We should draw the reader's attention to the fact that the original variables associated with the stationary point are not in the least constant; indeed their growth rates can vary both with time and depending on the values of other variables. Overall, only the rate of turnover of stocks and the degree of shortage must be constant over time. The stationary point is a particular kind of non-Walrasian dynamic equilibrium, a state of the system in which the "degree" of deviation from the shortage-free Walrasian state has stabilized and a chronic state of "normal" shortage is maintained. In what follows we call the path which satisfies (23) (that is, always remains at the sta-

tionary point) the *constant shortage path*.

3.1. Parameters not varying over time

By way of introduction it will be convenient to study systems with *constant parameters*, that is, where:

$$\tilde{a}(t) = \tilde{a}, \tilde{r}(t) = \tilde{r}, \tilde{b}(t, i) = \tilde{b} \text{ and } \tilde{h}(t) = \tilde{h}. \quad (24)$$

From the analysis of the function $g(v)$ given in the Appendix it turns out that there exists a quantity:

$$\tilde{h}_c = \max_v g(v) \quad (25)$$

which we call the *critical consumption ratio*. The significance of this terminology is shown by the fact that, if the consumption ratio exceeds this critical value, then according to (20), $v - v^2(\tilde{h} - \tilde{h}_c) > 0$; hence for an arbitrary initial value, $v(0)$, the system is sooner or later *unviable*: $v(t_1) = v_m$. In such a case, of course, there is no *stationary state*.

In contrast, if the *consumption ratio is less than the critical value*, then there exist *at least two* stationary states, since $g(0) = -\infty = g(v_m)$, so to the left and right of the maximum there is at least one stationary point (i.e., at least one on each side of the maximum).

Lemma 3. The equation $g(v) = \tilde{h}$ has at most two roots. The proof is given in the Appendix.

From what we have established so far the theorem below follows directly.

Theorem 1. The number of stationary states in a system with constant parameters is 0, 1, or 2, depending on whether the consumption ratio, $\tilde{h}$, is larger than, equal to, or smaller than the critical value, $\tilde{h}_c$.

Notes. For reasons that will become clear later on, we denote by $\bar{v}_s$ (stable) and $\bar{v}_u$ (unstable) the *smaller* and *larger* stationary points, respectively. We should mention that $\bar{v}_u(v^*)$ is an *increas-*

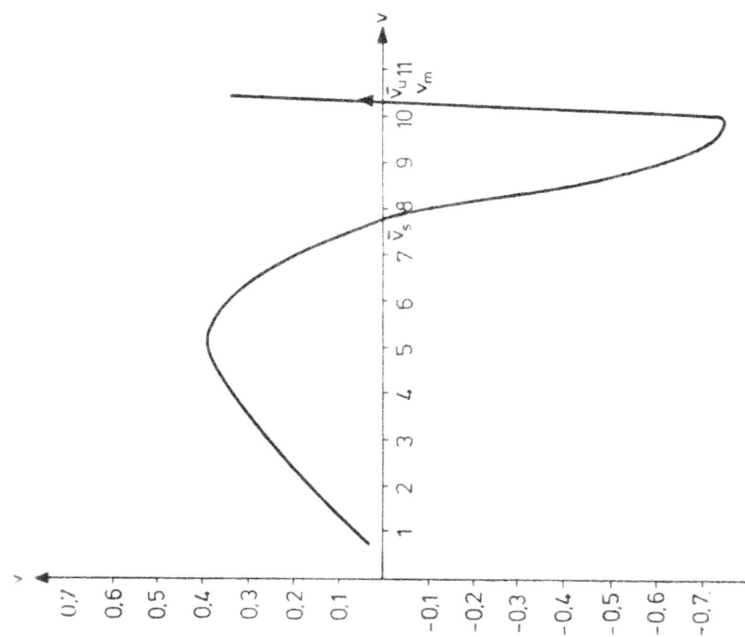


Figure 1. Acceleration of stock turnover as a function of the rate of turnover of stocks, and the two stationary points.

ing function while $\bar{v}_s(v^*)$ is decreasing. Moreover, it can be seen that there exists a pair of numbers v_u^* and v_s^* , for which $\bar{v}_u(v_u^*) = v_u^*$ and $\bar{v}_s(v_s^*) = v_s^*$.

Figure 1⁹ shows the acceleration of stock turnover as a function of the rate of turnover, based on relationship (20). The stationary points are the intersections of the curve and the horizontal axis.

Let us now turn our attention to an examination of the stability and viability of paths not starting at stationary states. From Figure 1 it is trivial to establish the following theorem.

Theorem 2. In a system with constant parameters, suppose $\bar{h} < \hat{h}$. Then paths beginning in the interval where $v_0 < v_m$ are stable

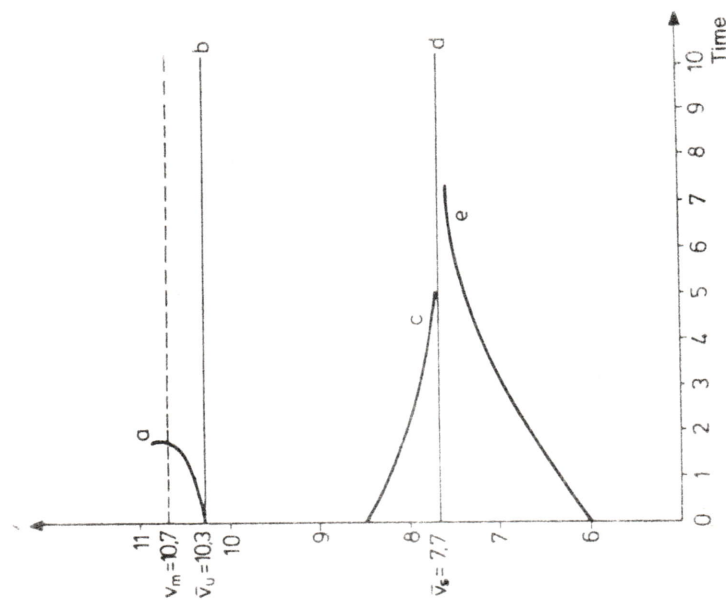


Figure 2. Stable and unstable paths. Notation: a = unstable and unviable path; b = unstable and viable path; c, d, and e = stable and viable paths

(they belong to $\bar{v}_s$) and viable; the path starting from $v_0 = \bar{v}_u$ ($v(t) = \bar{v}_u$ for all t) is unstable and viable; finally, paths starting on the interval $\bar{v}_u < v_0 < v_m$ are unstable and unviable (see Figure 2).

In a similar manner to theorem 2, the following two theorems can be proved.

Theorem 3. In a system with constant parameters, suppose $\bar{h} = \hat{h}$. Then $\bar{v}_u = \bar{v}_s = \bar{v}$ and paths starting on the interval $v_0 \leq \bar{v}$ are stable and viable, while those starting on the interval $\bar{v} < v_0 < v_m$ are unstable and unviable.

Theorem 4. In a system with constant parameters, suppose $\bar{h} > \hat{h}$. Then there does not exist a stationary state,

and all paths are *unviable*.

3.2. Systems with parameters varying over time

Let us now turn to an investigation of systems with *parameters varying over time*. Then it is unlikely that the roots of equation (23) would remain constant. From a theoretical standpoint, however, it is worth studying this special case. A trivial, but still useful, result is the following:

Theorem 1'. A state $\bar{v} < v_m$ is a *stationary state* if and only if for all t the *consistency condition*

$$\bar{h}(t) = g(t, \bar{v}). \quad (26)$$

is satisfied.

Notes. The following question may be raised: when does condition (26) give us a *stable* stationary state? Before answering the question we must introduce some notation and an assumption. For given $t \geq 0$ we denote the maximum of the function $g(t, v)$ by $\hat{v}(t)$. Then in the model with time-varying parameters, the critical consumption ratio can be represented as follows:

$$\hat{h}(t) = \max_v g(t, v) = g[t, \hat{v}(t)].$$

To exclude trivial cases we assume that $\hat{h}(t) > 0$. Consider the following supposition: whether there exists $\bar{v} \in (0, v_m)$ which is less than $\hat{v}(t)$ for all t :

$$v(t) > \bar{v} \quad (27)$$

It is apparent that when the condition $\bar{v} > \bar{v}$ is satisfied, then condition (26) gives a *stable* stationary state: $\bar{v} = \bar{v}_s$. Of course, $\bar{v}_s(t)$ that satisfies condition (26) generally depends on time, and for this reason the parallel result to theorem 2, set out below, can

only give us somewhat weaker results about viability and stability. In the case of time-varying parameters, the results below corresponding to theorems 2 and 3 can be established, assuming that (26) is satisfied.

Theorem 2'. Let us assume that the consumption ratio is always *strictly less* than the corresponding critical value: $\bar{h} < \hat{h} - \epsilon$, where ϵ is an arbitrarily small constant. Then there is a constant δ with the property that paths starting on the interval $v_0 < \delta$ are *stable and viable*.

Theorem 3'. Let us assume that the consumption ratio is always equal to its critical value: $\bar{h} = \hat{h}$. Then paths starting on the interval $v_0 > \bar{v}$ are *stable and viable*, while those starting where $v_0 = \bar{v}$ are *unstable and often unviable*.

Before we turn to a statement of theorem 4', we should underline the point that the condition given there cannot be satisfied together with (26).

Theorem 4'. Let us suppose that the consumption ratio is always *strictly greater* than its corresponding critical value: $\bar{h} > \hat{h} + \epsilon$, where $\epsilon > 0$ is an arbitrarily small constant. Then there is *no stationary state* and all paths are *unviable*.

Remarks. The principal qualitative result of the analysis is that in the system with time-varying parameters there can also exist viable and stable paths with constant shortage; these are states of the economy in which the "degree" of departure from the Walrasian equilibrium, the intensity of chronic shortage, has settled down to a steady level. The system is able to regulate itself endogenously in such a way that having moved away from the path of constant shortage, it would return. The degree of shortage is an adequate signal for effecting this regulation; or, alternatively, the rate of turnover of stocks can be used.

At the same time we should draw attention to the fact that we have actually stated an existence theorem for paths with constant shortage; moreover, we have given the conditions for the existence of such paths, but we have by no means clarified the necessary and sufficient conditions for their stability and viability. With time-varying parameters, not only are we unable to deter-

mine the state separating viable and unviable states, but even the existence of such a separating state is uncertain.

Likewise, we still have to complete the analysis of *mixed cases*, when for instance $\bar{h} < \hat{h} - \epsilon$ and $\bar{h} > \hat{h} + \epsilon$ are satisfied in turn, with intermittent cycles.

4. Alternative growth strategies: a comparison

While in previous sections we established essentially qualitative results about the existence, stability, and viability of a stationary point (paths with constant shortage), now we take a step toward the economic application of our apparatus. We do not seek to carry out a numerical, econometric examination of the real, historical, or possible future development of some one country or another. Rather, we remain at the level of *pure theory*. We introduced some supplementary assumptions aside from the earlier assumptions and show what their consequences are. For illustrative purposes we carried out some simulation calculations, using stylized Hungarian data.¹⁰ We should emphasize that these data serve exclusively to illustrate our theoretical conception and should not be considered as a realistic reflection of actual historical development.

In the Eastern European socialist countries over the last decade there has been a trend toward lower efficiency; in what follows, we try to survey the exogenous and endogenous factors involved. We regard as exogenous that decline in efficiency that would occur in the system without shortage. In our calculations we formalize this notion as follows:

$$\tilde{a}(t) = \tilde{a}_0 + (-1/\tilde{a})t \quad (28)$$

and

$$\tilde{c}(t) = \tilde{c}_0 + (-1/\tilde{c})t. \quad (29)$$

Comparison of Alternative Growth Strategies

	Strategy symbol		
	A	B	C
Properties of the strategies			
Exogenous factors			
Cost function	increasing $a^A(t, Z)$	increasing $a^B(t, Z)$	increasing $a^C(t, Z)$
Investment efficiency function	decreasing $b^A(t, i, Z)$	decreasing $b^B(t, i, Z)$	decreasing $b^C(t, i, Z)$
Strategic factors			
Maximum net investment ratio	constant $\tilde{i}_n^A(t) = \tilde{i}_0$	increasing $\tilde{i}_n^B(t)$	increasing $\tilde{i}_n^C(t)$
Net consumption ratio	growing slightly $\tilde{h}_n^A(t) \sim \tilde{h}_n^A(0)$	decreasing $\tilde{h}_n^B(t)$	growing slightly $\tilde{h}_n^C(t) \sim \tilde{h}_n^A(t)$
Degree of shortage	constant $Z^A(t) = Z(0)$	constant $Z^B(t)$	increasing $Z^C(t)$
Consequences			
Viability	always	always	temporarily
Capacity growth	slow $K^A(t)$	less slow $K^B(t)$	slow $K^C(t) \sim K^A(t)$
Growth of net output	slow then stops	slow	rapid, then falls
Growth of consumption	proportional	temporary	temporary

It will prove convenient to shift to using *net* production, and the corresponding net investment ratio, instead of the gross output and gross investment ratio used before. Identifying the net quantities with a suffix n , we can express them in terms of the gross figures:

$$\tilde{i}_n = \tilde{i}/(1 - \tilde{a}), \quad \tilde{i}_n = \tilde{i}/(1 - a), \quad X_n = (1 - a)X. \quad (30)$$

Along a path with constant shortage the four investment ratios, $\tilde{i}_n, \tilde{i}_n^A, i$, and i_n move in proportion to one another, but on other

paths they change in relation to each other in a more complicated way.

Let us turn to an analysis of the endogenous factors involved in a decline in efficiency. In agreement with our macro-perspective we neglect structural questions, and in the first place we use the increasing intensity of shortage to explain both the increase in the cost coefficient and the decline in the investment efficiency, aside from the effect of exogenous factors.

We compare three characteristic growth strategies, which differ from one another in just one strategic factor (investment or consumption ratio).¹¹ Let us suppose that the external conditions and the initial values for all three paths are the same.

Table 1 gives the characteristics of the three strategies, and Figures 3, 4, and 5 show the growth paths.

Let us now briefly refer to the similarities and differences between the three strategies.

Strategy A sticks to the initial net (maximum) investment ratio despite the worsening production and investment strategy. It resigns itself to a reduced rate of capacity growth, but maintains the initial intensity of shortage and the consumption ratio consistent with that. According to the consistency condition (26) of theorem 1' the consumption ratio is essentially constant. *Strategy A* ensures relatively balanced growth: national income and consumption grow in proportion to one another, albeit at a slow rate.

Strategy B seeks to avoid accepting such a slowdown in growth and by raising the net investment ratio at least tries to brake the decline in the growth rate. However, this strategy, too, sticks to the initial intensity of shortage, and in accord with the consistency condition the consumption ratio therefore declines. Not surprisingly, in strategy *B* both capacity and national income grow more rapidly than in *A*; however, consumption not only lags behind that of *A*, but the initial weak growth of consumption turns into a decline.

Strategy C simultaneously seeks to ensure the consumption policy of strategy *A* (using the maximum net consumption ratio

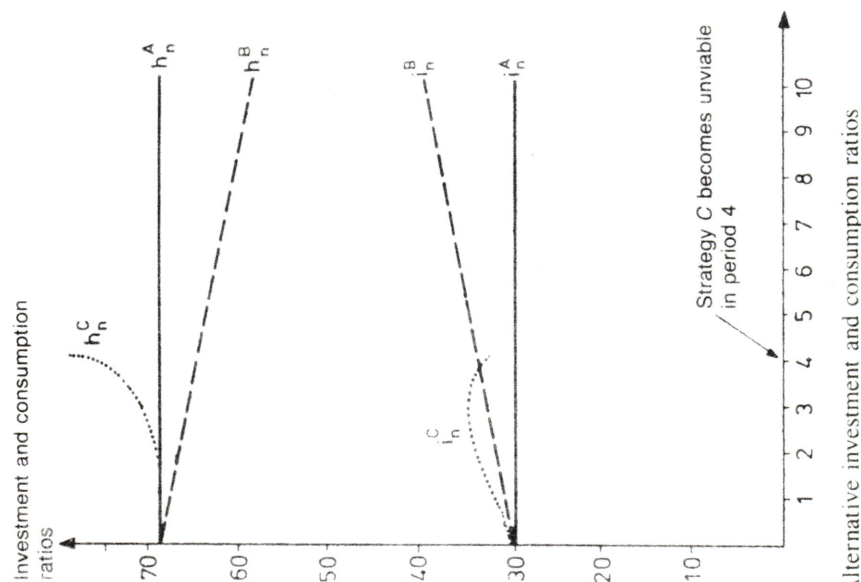


Figure 3. Alternative investment and consumption ratios

from strategy *A*) and the investment policy from strategy *B* (using the maximum net investment ratio of strategy *B*). This policy violates the consistency condition (26) and uses up the stocks more and more: the rate of turnover of stocks and the degree of capacity utilization gradually approach their upper limits, and the system becomes unviable. Because $a(t)$ grows more rapidly than $a(t)$, the net and gross, or alternatively the maximum and actual indicators, display divergent trends. Let us draw attention to the rapid initial growth of net output which is dependent on increasing capacity utilization; soon, however, the worsening efficiency brought about by shortage turns this into a decrease.

Approaching the end of our work, we should like to make the following remarks. To understand the growth problems of a socialist economy it is not sufficient merely to use a simple Keynesian or neoclassical growth model. Even in a system with parameters not varying over time one cannot confine oneself to determining the *maximum* growth rate (as for instance Phelps [18] does in his classic work), while neglecting the large and significant effects of shortage. So we believe that our study of development paths along which the growth rates of investment and consumption can diverge, instead of growth theory with proportional paths, significantly increases the realism of our model.

We still have to say something about foreign trade and a more detailed investigation of consumption. As we mentioned earlier, however, these will be tasks for later articles.

Appendix: proofs of lemmas

Proof of lemma 1

Because of the smoothness conditions on the parameter functions, the differential equation (20) has a unique solution for any initial value on the interval $(0, v_m)$.

Proof of lemma 2

To begin with, let us suppose that the model has at least one solution. From this assumption we can deduce the fundamental equation (20) Let us substitute into the relation $V/K = k/v$ the relation (14).

$$V/K = k^*/(v^{**}v^{1-\alpha})$$

Solving this for v , we have:

$$v = (k^*K/v^{**}V)^{1/(1-\alpha)}$$

Now differentiate v with respect to time and take account of the relationships $V/K = (1 - a - b)k$ and $K/K = r$, which follow from (1) through (8). After some calculation—substituting into equations (16) and (19)—we get equation (20).

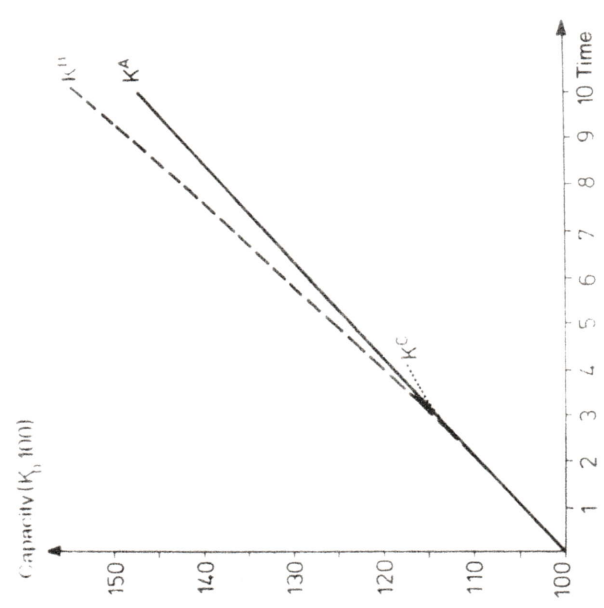


Figure 4. Alternative dynamics of capacity

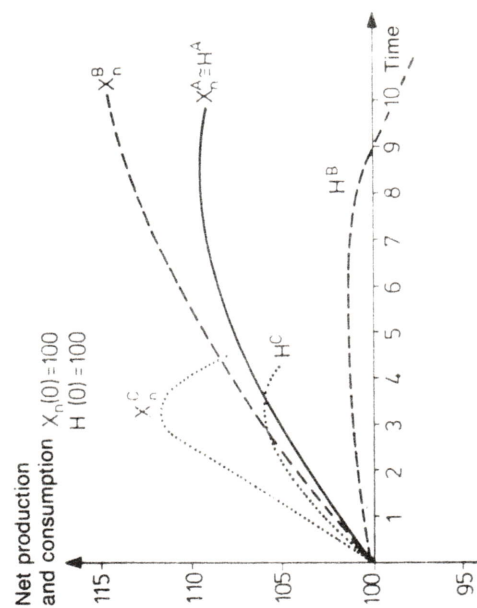


Figure 5. Alternative net output and consumption paths

For a given initial state (V_0, K_0) in the above terms, the initial state v_0 can be determined uniquely. Let us therefore solve the fundamental equation by starting from the calculated initial value v_0 ; given a knowledge of v , we can then determine the value of Z using (17) and the values of a, b, i , and k using (16). Finally, r is determined from (18).

We begin the process of getting back to absolute values by using the formula $K(t) = K_0 \exp \int_0^t r(s) ds$. Then X is determined from formula (4), V from (3), A from (5), I from (7), B from (6), and H from (8).

Proof of lemma 3

For convenience we make the transformations $w = 1/v$ and $Q(w) = 1/Z(v)$. Multiply the expression $g(w) = \bar{h}$ by Q^α and denote by $G(w)$ the resulting expression:

$$G(w) = \bar{a} - (1 - \bar{h})Q(w)^\alpha + \bar{i}Q(w)^{\alpha+1} + r^*w^{\alpha\kappa}w^{1-\alpha}Q(w)^{\alpha+\beta+\theta}.$$

Our proof can refer back to known results [19, volume 2, part 5, exercise 77], if $\alpha = 1$. (Note: this parameter value is not actually permitted!)

When $0 \leq \alpha < 1$, however, it seems simpler to give a direct proof rather than try to refer the problem back to results from the above source. Our proof is as follows:

Let $G(w)$ have at least three roots on the interval $w < w_m$: $\bar{w}_1 < \bar{w}_2 < \bar{w}_3$. By a simple transformation we get the following relationship:

$$1 - \bar{h} = \bar{i} \cdot \frac{Q(\bar{w}_2)^{\alpha+1}}{Q(\bar{w}_1)^\alpha} \frac{Q(\bar{w}_1)^{\alpha+1}}{Q(\bar{w}_1)^\alpha} + r^*w^{\alpha\kappa} \frac{\bar{w}_2^{1-\alpha}Q(\bar{w}_2)^{\alpha+\beta+\theta}}{\bar{w}_1^{1-\alpha}Q(\bar{w}_1)^{\alpha+\beta+\theta}} \frac{Q(\bar{w}_1)^{\alpha+1}}{Q(\bar{w}_1)^\alpha}.$$

We now use Cauchy's elementary mean value theorem, applying it separately to the two terms on the right-hand side: thus there exist numbers w_{12} and $\hat{w}_{12}$ on the interval $(\bar{w}_1, \bar{w}_2)$ such that the terms are equal to the ratios of derivatives of numerator and denominator evaluated at w_{12} and $\hat{w}_{12}$ respectively, since $[Q(w)^\alpha] = \alpha Q(w)^{\alpha-1}Q'(w)$, and so on. Instead of our equality we can write the following expression:

$$1 - \bar{h} = \bar{i} \frac{(\alpha + 1)Q(w_{12})}{\alpha} + r^*w^{\alpha\kappa}w_{12}^{1-\alpha} \frac{(\alpha + \beta + 1)Q(\hat{w}_{12})^{\beta+\theta}}{\alpha} + r^*w^{\alpha\kappa}(1 - \alpha) \frac{\hat{w}_{12}^{1-\alpha}Q(\hat{w}_{12})^{\beta+\theta+1}}{Q'(\hat{w}_{12})}.$$

Similarly we can justify the existence of a pair of numbers $(w_{23}, \hat{w}_{23})$, both belonging to the interval $(\bar{w}_2, \bar{w}_3)$ and satisfying the last relationship. That is all very well, but the first and second elements are clearly increasing functions and the third can be shown to be after some calculation. In fact,

$$\frac{Q'(w)}{w^{1-\alpha}Q(w)} = \frac{k^* \left(\frac{w^*}{w} \right)^\alpha}{k_m} \left[1 - \frac{k^* \left(\frac{w^*}{w} \right)^{\alpha-1}}{k_m} \right]^{-1} \frac{k^* \left(\frac{w^*}{w} \right)^{\alpha-1} (\sigma - 1)w^*}{w^{2-\alpha}} + (1 - \zeta) \left[1 - \left(\frac{w_m}{w} \right)^\alpha \right]^{-1} \tau \left(\frac{w_m}{w} \right)^{\alpha-1} \frac{w_m}{w^{2-\alpha}}$$

and this is indeed a decreasing function.

But then we have a contradiction, since $(w_{12}, \hat{w}_{12})$ and $(w_{23}, \hat{w}_{23})$ cannot both satisfy the above relationship simultaneously.

Notes

1. For a survey, see for instance [5] as well as the introduction by Sen to his edited volume [21]. On the theory of growth of a socialist economy the classical model is Kalecki [11]. Some attempts have been made to link together the traditional growth theory and the disequilibrium theory [7]. Our own approach has some similarity with this French line of research. In the disequilibrium theory, Honkapohja and Ito [8] emphasize the role of stocks, Malinvaud [16] the role of investment. Of the static models concerned with the relationship between the disequilibrium theory and socialist economies we should mention Portes [20].

2. Apart from the works mentioned above, the following works from the Hungarian literature provide foundations for the present work: Kornai and Martos [14] on non-price control, Bauer [2] on the theory and practice of

investment cycles in the socialist countries, and Lackó [17] on an econometric study of the investment process.

4. Our conventions about notation are these. We denote the *absolute* variables of the model with uppercase [italic] Latin letters, the corresponding *relative* variables with the same lowercase letter. Parameters in the *system without shortage* are distinguished from those in the *actual* system by the sign $\sim$. In general we denote *constants* (e.g., elasticities with respect to shortage) by lowercase Greek letters, though norms and extreme values of variables regulated by norms are exceptions: these—although also constants—are distinguished by asterisks or subscripts from the corresponding variables denoted by lowercase [italic] Latin letters.

4. The *ordinal* index Z is related to the "degree of shortage" but it is not the same as the *cardinal* notion of aggregate excess demand used in standard theory. Because of space constraints we cannot undertake to elaborate here on why we prefer this index Z to the usual aggregate excess demand (on this, see Kornai [13, pp. 20-32, 41-46, and 152-155]).

5. Theoretically, it would be more correct to modify the form of the function (9) representing Z so that $Z = 1$ would correspond to positive v and k . Otherwise, a suitable choice of the parameters σ and τ makes it possible for Z to be approximately equal to 1 for realistic values of v and k . In our computations, such parameters did occur.

6. In the growth theory literature, the following alternative assumptions are known. In the Harrod-Domar model $\omega = 0$, inasmuch as the growth rate is less than the "natural" rate. In the neoclassical growth model [18], $\omega = 1$. We should mention that Eltis [4] chose a function of the form $\bar{b}(i) = \bar{c}_0 + \bar{c}_1 i$. (All the authors just mentioned worked with models assuming constant efficiency of investment.)

7. This—within the framework of the present non-linear model—is similar to the control rule in models with linear stock signals in Kornai and Martos [14].

8. It is clear that rule (14) associates the highest capacity utilization with the most rapid rate of turnover of stocks, v_m . Naturally, it can turn out that the capacity utilization obtained in this way, $k^*(v_m/r^*)^\alpha$, is larger than the maximum permitted capacity utilization, k_m . Then it is necessary to modify the v constraint from v_m to $v_k = v^*(k_m/k^*)^{1/\alpha}$. In what follows, we assume that $v_k = v_m$ so that v_m remains the upper limit.

9. In constructing Figures 1 and 2 we used essentially the same data as in the simulation referred to in section 4 (see also footnote 10).

10. In general we took Hungarian macro-data for the 1970s as our basis, apart from the shortage parameters. These latter had to be given assumed values. The diagrams represent paths calculated in the simulations. We have not reported our data because the calculations are merely attempting to illustrate theoretical relationships.

11. In her numerical study of long-term planning, Augusztinovic [1] also considers alternative growth strategies: MAXIMOST is similar to our A strategy, while MAXIMAJD focuses rather more on raising the investment ratio than does our strategy B .

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Kedves Rita,

Írom a cím fordítását:

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Vajon Orbán Viktor uralma a végtelenségig fog tartani? A történelmi tapasztalatokból arra lehet következtetni, hogy az autokratikus hatalomnak csak egy földrengés vethet véget."

Üdvözlettel,

Weronika

2015. március 3. 14:14 Fancsovits Rita írta, <rita.fancsovits@uni-corvinus.hu>:

Kedves Weronika,

köszönjük a fordítást. Professzor úr is megnézi és a jövő héten fogom utalni a pénzt. Szerdán, mert addig szabadságon leszek. Remélem, ez megfelel.

A véletlen úgy hozta, hogy közben még egy lengyel vonatkozású