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AUTONOMOUS CONTROL OF THE ECONOMIC SYSTEM

BY JÁNOS KORNAI AND BÉLA MARTOS[1]

The possible impact of autonomous control (vs. the price mechanism and directive control) on the functioning of an economic system is studied. It sheds some light on similarities among economic systems that are quite different in their higher functioning. The survival and stability conditions for a Leontief-type economy demonstrate the role stock signals can play in controlling the behavior of producers and consumers.

COMPLICATED SYSTEMS FULFILLING a number of duties are controlled, generally, by multi-stage regulators consisting of both simple and complex mechanisms. For example, some of the functions of the spaceship are controlled by simple, built-in servomechanisms; others are guided half-automatically from the earth; and still others are guided directly by handpower of the astronauts. Or let us take a higher living organism, the human body, say. Some of its functions, respiration, digestion, blood circulation, functioning of the heart, lungs, stomach, intestines, and kidneys are controlled by the autonomous (vegetative) nervous system; other functions by the central nervous system.

An economy is also a complicated system, fulfilling a great number of functions. Its processes are controlled by several kinds of mechanisms of low and high degree. The low degree mechanism, following the physiological analogy, is called autonomous (vegetative) control.[2]

The first section of our study explains the idea of the "control mechanism," draws comparisons between the different mechanisms, and presents some empirical findings. Sections 2 and 3 analyze the autonomous control theoretically, with the aid of a general and a special model. In the fourth section we draw conclusions and comment on the previous theoretical analysis.

1. CONTROL MECHANISMS

1.1. *Price Mechanism and Directive Mechanism*

For an abstract analysis of the functioning of the economic system we divide it into two spheres. The material and physical processes of the economy, such as production, displacement of products, and consumption, take place in the *real sphere*. The *control sphere* is to guide the real processes. It is here that the gathering,

[1] This study is based on the ideas of economics and model building which J. Kornai set out in his book [4] and his paper [5]. The further development of his general model and the special model presented here are the common work of the authors; the analysis of the latter was done by B. Martos.

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[2] In the English language physiological literature, the adjective "autonomous" is used to indicate the low degree nervous system. Other languages, among them Hungarian, use the word "vegetative." The analogous economic phenomenon we are presenting here is well described by both adjectives.

processing, and transfer of information take place, as well as the preparation and making of decisions.

Within the control sphere we discern subsystems which we name *control mechanisms*. These, although taking effect in close connection with each other, are independent to a certain degree.

Control mechanisms can be distinguished mainly by the following criteria (cf. [**1**]):

(i) *Organization*. Which organizations of the economy take part in the functioning of the mechanism?

(ii) *Information structure*. Which types of information are characteristic of the flow of information in the mechanism? Which organizations are connected by information channels within a given mechanism?

(iii) *Rules of behavior*. Which regularities are characteristic of the decision-making processes of the organization in the mechanism? To the different types of information input, which information output is their answer, i.e., which response function will describe their behavior?

(iv) *Motivation*. Which motives make the organizations form their rules of behavior and decision, and the corresponding information structure?

In what follows we summarize, in very simple outlines, the criteria of two pure control mechanisms. First comes the *price mechanism*. Its structure is entirely decentralized; all producers and consumers, buyers and sellers take part in it. It contains no legally sanctioned sub- and superordination. The main characteristic of its information structure is that it is the price that serves as a signalling system for the control. In case of oversupply the price goes down; in case of excess demand the price goes up. The rule of behavior of the producer is that if the price of some product rises, more has to be produced and less used up (and if the price goes down, the case is reversed). The motivation of the producer is to increase his profit. (We omit discussion of the behavior and motivation of the consumer.)

The second mechanism to be mentioned here is central control through instructions, in short, the *directive mechanism*. Its structure is fully centralized and hierarchical. All real activities in an economy are directed ultimately, but usually not directly, by the center. Under the center and above the producing firms, there are sub-centers and middle grade directing organizations placed on one, two, or several levels. Characteristics of the information structure is the "vertical" flow of information. From the top instructions are sent downward, and from below reports are sent upwards. Relying on the reports the center makes decisions. The rule of behavior serving to control short-term adaptation is that if the reports reveal shortage, instruction is given to increase production; if a surplus is shown, the instruction is to reduce production. Motivations of the producing firms are discipline and material and moral interest in following instructions. (We disregard here again the behavior of the consumer.)

An important part of the literature and mainly neoclassical price theory creates the impression by its sophistication that the real sphere of the capitalist economy is controlled solely by the price mechanism.[3] A similar one-sidedness can be

[3] Although there are, fortunately, other theoretical lines of thought (theories of imperfect competition, behavioral theory of the firm, etc.) which draw attention to further control and information processes.

observed in works dealing with the socialist economy. The latter suggest, conversely, that the socialist economy (at least in the period prior to the decentralization reforms) is solely controlled by directive mechanism. These one-sided characterizations however are not relevant. In both capitalist and socialist economies several kinds of control mechanisms work simultaneously. In the capitalist economy there appear, although not covering the whole system but only some part of it, the elements of the directive mechanism, in the direction of the big concerns and the national economic organizations. On the other hand, a price mechanism already worked in the socialist economy before the reforms, even if in narrow ranges, and its power has grown following the reforms.

The control mechanisms described above are supplemented by further ones, such as the medium and long term planning of the national economy, which has had an important effect in socialist countries from the start and is gaining importance in developing countries and even in advanced capitalist countries.

1.2. *Autonomous Control*

A further controlling mechanism is *autonomous control*. This is, in fact, a complex regulator consisting of several sub-mechanisms. *It is to be found in both capitalist and socialist economies.*

The most important component, i.e., sub-mechanism of the autonomous control mechanism, dealt with in detail in this study is the *control based on stock signals*. One of the most important sources of information for the producing firm is the observation of the stocks of its own products and of the materials it uses. If there is an overstock of its own products, it is expedient to reduce the production level, and if the stock has shrunk, it is desirable to increase production. Similarly, if the stock of material has grown, purchase can be reduced, and if it is too small, purchase should be increased. In the consumer's household, decisions on purchases can be made using the same logic.

Another component of the autonomous control mechanism is the *direct connection between seller and buyer*. They inform each other by offers, advertisements, preliminary discussions, and orders about what they can or want to sell or buy.

It is characteristic of autonomous control that it always takes place on the lowest level, between producers and consumers, without the intervention of higher administrative organizations. It is autonomous, i.e., not connected directly to any complicated general social process. The directive mechanism is based on a precise teamwork of several hundreds of firms and offices and on a complicated flow of instructions and reports. In the price mechanism the price is developed mutually by the participating firms and households. In autonomous control, this process is much more of a local character. In control based on stock signals, the firm or household watch their *own* stock only; here we deal with the process of information and decision-making taking place *within* one single cell of the organization. In the direct connection between buyer and seller two "neighbouring" cells of the organization meet.

A further characteristic of autonomous functioning is its simplicity—we might say its primitive nature. Here we are talking about trivial phenomena known to everybody; only theory has left it out of consideration so far. The motivation of autonomous control may be found, probably, along the following lines. Every organization that has been established tries to live on. The overwhelming majority of the persons belonging to it identify themselves with the organization even if only to the extent that they want it to survive, and to function permanently and smoothly, at least if that does not require more than the usual effort. In this way the buyer of materials, the production engineer, and the employee of the sales department identify themselves to a certain extent with their sphere of activity and feel it natural that it is their duty to supply the firm with material continuously, to run the shop smoothly, and to sell products. They all try to fulfil their task as best they can to make the organization run smoothly. There must be other motives to autonomous functioning, but the one described above seems the most important.

Just as the elementary functioning of the living organism (blood circulation and respiration, for example) is not controlled by conscious acts of the central nervous system, but by the autonomous nervous system, the elementary functions of the economic system are guided by autonomous control. It is not true that a capitalist firm will in all cases wait until the *price* of product A goes down and that of product B goes up to reduce the manufacturing of product A and increase that of product B. It is not true either that a socialist firm will wait for *instruction* to reduce the manufacturing of product A and to increase that of product B before it sets out to do it. Autonomous movements arise in both societies. The firm autonomously reduces the manufacturing of the product A and increases that of product B because it sees that the stock of the former is overgrown and that of the latter has shrunk; or, more simply, the buyers declared that they wanted less of the product A and more of the product B.

In Table I we sum up the criteria of the controlling mechanisms that have been described. We shall make a few comments here on the table.

The control and information processes described in columns 2 and 3A are in many cases closely interrelated. In the buyer's and seller's offers and contracts information of a price and non-price character usually is to be found all together. Yet we feel it right that the controls described in columns 3A and 3B should be considered *together* the *third* mechanism, i.e., the autonomous control of the economy. The main criterion of delimitation here is the following: the importance and role of the first and second mechanisms are heavily dependent on the concrete nature of the economic system, while those of the third mechanism are almost independent of it.

The following question now arises: where do we place the concept of the "market mechanism" in the conceptual framework introduced here? Literature uses many kinds of definitions; therefore, it would not clarify the situation if we adopted one of them arbitrarily. We shall content ourselves with a less concrete circumscription. The common interpretation of "market mechanism" implies, so to speak, everything that is included in columns 2, 3A, and 3B of our table.

TABLE I

CRITERIA OF THE CONTROL MECHANISMS

	1 Directive mechanism	2 Price mechanism	3A Autonomous control mechanism Direct connection between buyer and seller	3B Stock signal
Organization	centralized, hierarchic	decentralized, result of the common activity of all buyers and sellers	decentralized, built exclusively on the connection of one pair of buyer and seller	decentralized, takes place exclusively within the organization
Information structure: main type of information	downwards: instruction upwards: reports	price	direct information on demands and possibilities	stock signal
Producer's rules of behavior	instruction → fulfilment	change of price → change of output and input	buyer's (seller) information → (consumer's) adaptation	change of stock → changing of output and input
Motivation of producer	discipline, interest in the fulfilment of instructions	increasing the profit	identifying themselves with the survival of the firm and with its smooth running	

Having arrived at the end of the first section treating the conceptual system, we wish to emphasize that we ourselves do not consider the definitions and classifications of concepts as introduced here satisfying and final in every respect. In any case, we feel the necessity of going beyond the rigid schemes of thought which, for a long period, have simply identified the mechanism of economic control with the price mechanism and economic information with price.

1.3. *Experiences*

Autonomous control can be observed empirically. In order to describe it we have to stay in the sphere of actual production and study closely the life of the firms, the information utilized, and the rules of thumb used in the decision-making process. The personal experiences of the authors, as well as the discussions they had with a number of managers, seem to support the existence of autonomous control and its extremely important power. However, for a methodical description of its functioning and of its connection with other control mechanisms, further empirical and theoretical research is needed.

The reform of Hungarian economic management and control provided an indirect proof of the important role of autonomous control. More than four years ago, on the 1st of January, 1968 central directive control was eliminated overnight in a very large field of the economy, at least in the sphere of short-term decisions. Since then, however, only a relatively small part of the real sphere has been controlled by a pure price mechanism, by prices adapting themselves to the equilibrium of the market. In spite of this, no vacuum has been created in economic management. Autonomous control existed before and after the reform. We are convinced that it is mainly on this account that production went on by itself and that the real processes of economy continued to take place rather smoothly.

However important further empirical research may be, there are limits to it. In practice, autonomous control is closely correlated to other, higher mechanisms. In a pure form, separated from the rest, it can only be analyzed theoretically with the aid of mathematical models.

2. GENERAL MODEL

In our present study we deal with the theoretical analysis of only one component or sub-mechanism of autonomous control, that based on stock signals. First we describe a general model (also called a meta model) which will serve as a framework for a whole series of specific research projects; then in Section 4 we will specify the general model and carry out analysis on a simple, concrete model.

In the economic system m primary limited resources are available and n products are manufactured. The system contains M consumers and $N + 1$ producers, producer 0 being Nature who "produces" primary resources from nothing.

The system is dynamic; all its variables are dependent on time, t. Time, as argument, will be written out only occasionally in what follows, in case we want to emphasize that some variable is a *given*[4] function of time (exogenous variable). Derivatives with respect to time will be indicated by a dot above the variable.

2.1. *Variables*

At the present general level of discussion we leave open the question of whether stochastic or deterministic variables will be shown in the model.

Endogeneous variables

u_{ij}: the ith producer's stock of the jth product (output stock) ($i = 1, \ldots, N$; $j = 1, \ldots, n$).

v_{ij}: the ith producer's stock of material (input stock) of the jth product ($i = 1, \ldots, N$; $j = 1, \ldots, n$).

w_{ij}: the ith consumer's stock of the jth product ($i = 1, \ldots, M$; $j = 1, \ldots, n$).

r_{ih}: the ith producer's stock of the hth resource ($i = 0, 1, \ldots, N$; $h = 1, \ldots, m$).

[4] The word "given" need not mean that the time function is known beforehand, i.e., at $t = 0$, but only that its value is observable at any point of time. The possibility of observation error remains open, but is neglected in what follows.

x_{ij}: the ith producer's production of the jth product ($i = 1, \ldots, N$; $j = 1, \ldots, n$).
y_{ikj}: the ith producer's purchase of the jth product from the kth producer ($i = 1, \ldots, N$; $k = 1, \ldots, N$; $j = 1, \ldots, n$).
z_{ikj}: the ith consumer's purchase of the jth product from the kth producer ($i = 1, \ldots, M$; $k = 1, \ldots, N$; $j = 1, \ldots, n$).
s_{ikh}: transfer of the hth resource from the kth producer to the ith producer ($h = 1, 2, \ldots, m$; $i, k = 0, 1, \ldots, N$).
p_{ij}: the ith producer's consumption of the jth product ($i = 1, \ldots, N$; $j = 1, \ldots, n$).
q_{ih}: that part of the ith producer's stock of the hth resource, which is put to use ($i = 1, \ldots, N$; $h = 1, \ldots, m$).

Exogenous variables

$g_{ij} = g_{ij}(t) \geqslant 0$: the ith consumer's consumption of the jth product ($i = 1, \ldots, M$; $j = 1, \ldots, n$).
$r_h = r_h(t) \geqslant 0$: full stock of the system of the hth resource ($h = 1, \ldots, m$).

2.2. *General Assumptions*

ASSUMPTION 2.1: The variables are continuously differentiable in the domain $t \geqslant 0$. Their initial value is given for the point of time $t = 0$. The initial value is marked by superscript 0 (e.g., x_{ij}^0, $u_{ij}^0, \ldots$).

ASSUMPTION 2.2: There are no joint products ($p_{ij} \geqslant 0$).

ASSUMPTION 2.3: Resources cannot be produced ($q_{ih} \geqslant 0$), but in the course of their utilization they are not used up. Their natural increase or decrease changes the 0th producer's stock.

ASSUMPTION 2.4: Quality of the jth product is homogeneous; it is independent of the producer making it. Accordingly, it is the same for all users, producers or consumers, whether the product comes from one or another producer. We assume that if supply covers demand, i.e., the balance equations (described later on) are satisfied, then a mechanism exists which will ensure that buyer and seller meet. This mechanism will not be modelled here.[5]

2.3. *The Model*

Input functions

(2.1) $$p_{ij} = p_{ij}(x_{i1}, x_{i2}, \ldots, x_{in}, t),$$

(2.2) $$q_{ih} = q_{ih}(x_{i1}, x_{i2}, \ldots, x_{in}, t).$$

We assume $p_{ij}, q_{ih} \geqslant 0$ if $x_{i1}, \ldots, x_{in}, t \geqslant 0$ (see Assumptions 2.2 and 2.3).

[5] The same assumption is implicit in the mathematical models of the price mechanism.

Balance equations

$$\dot{u}_{ij} = x_{ij} - \sum_{k=1}^{N} y_{kij} - \sum_{k=1}^{M} z_{kij}, \tag{2.3}$$

$$\dot{v}_{ij} = \sum_{k=1}^{N} y_{ikj} - p_{ij}, \tag{2.4}$$

$$\dot{w}_{ij} = \sum_{k=1}^{N} z_{ikj} - g_{ij}, \tag{2.5}$$

$$\dot{r}_{ih} = \sum_{k=0}^{N} s_{ikh}, \qquad i \neq 0, \tag{2.6}$$

$$r_h = \sum_{i=0}^{N} r_{ih}, \tag{2.7}$$

$$y_{ikj} = -y_{kij}, \tag{2.8}$$

$$s_{ikh} = -s_{kih}. \tag{2.9}$$

Equations (2.3)–(2.9) give the trivial balance conditions of production and turnover. With equation (2.6) we took Assumption 3 into account.

Rules of behavior

$$x_{ij} = \xi_{ij}\left[u_{ij}, \sum_{k=1}^{N} y_{kij} + \sum_{k=1}^{M} z_{kij}\right], \tag{2.10}$$

$$\sum_{k=1}^{N} y_{ikj} = \eta_{ij}[v_{ij}, p_{ij}], \tag{2.11}$$

$$\sum_{k=1}^{N} z_{ikj} = \zeta_{ij}[w_{ij}, g_{ij}], \tag{2.12}$$

$$\sum_{k=0}^{N} s_{ikh} = \sigma_{ih}[r_{ih}, q_{ih}]. \tag{2.13}$$

Production and the purchase of products and resources are functions of stocks and consumption. Here the functions $\xi_{ij}[\cdot,\cdot]$, $\eta_{ij}[\cdot,\cdot]$, etc. are to be interpreted as operators; they may contain, e.g., derivatives with respect to time, integrals, and other operators too. It should be observed that no behavioral rule needs information from an outside source.

2.4. *Raising the Theoretical Problem: Conditions of Survival*

The issue, in its most general form, is the following: *Is it possible to establish rules of behavior based on stock signals which will ensure the survival and permanent functioning (or even growth) of the system?*[6]

In order to answer the question, we have to define the notion of "survival" or "permanent functioning." In the economy described by the system (2.1)–(2.9) a weak notion of survival can be defined by the following inequalities:

(2.14) $x_{ij} \geqslant 0,$

(2.15) $u_{ij}, v_{ij}, w_{ij} \geqslant 0,$

(2.16) $r_{ih} \geqslant q_{ih}, \qquad i \neq 0,$

(2.17) $r_{0h} \geqslant 0.$

That is, production and the stocks of products cannot be negative, and utilization of the resources cannot surpass their capacity. Of course, stricter conditions of survival than these can also be prescribed.

The question can only be answered if the general model is specified to a certain extent. We require further assumptions regarding input functions, consumption, and resources. In the section that follows we shall present such a strongly specified variant of the model.

3. SPECIAL MODEL: LEONTIEF ECONOMY, *PI* CONTROL[7]

3.1. *Special Assumptions*

In addition to the four general assumptions introduced in the previous section, we now introduce further ones. The first group of these (3.1–3.3) concerns the structure of the economy.

ASSUMPTION 3.1: All the variables are deterministic and their value is observable without error.

ASSUMPTION 3.2: There is only one consumer ($M = 1$).

ASSUMPTION 3.3: The real sphere of the economic system is of the Leontief type, i.e., (i) there are no external limited resources ($m = 0$); (ii) the ith product is made by

[6] In this article we do not wish to raise the question of which conditions would make the functioning of the system optimal. In our opinion, the study of optimality does not belong to the construction of the abstract descriptive theory of the problem. Accordingly, as indicated in Section 4, it will not belong in our future research program either. A detailed description of our view on optimality is to be found in [**4**, ch. 10 and 11].

[7] In terms of the theory of automatic control (see e.g. [**6**]), this means a feedback control containing both a proportional (*P*) member and an integrator (*I*). The load (in our case, consumption and sales) will also be fed back.

only one producer, the ith sector ($n = N$)[8]; and (iii) inputs are proportional to production:

$$p_{ji} = F_{ij}x_j. \tag{3.1}$$

The following group of assumptions (3.4–3.7) stipulates certain interrelations among the $F(t)$ input-coefficient matrix, the $g(t)$ consumption function, and the initial values x^0, u^0, V^0, w^0, which are all assumed given. These conditions, as we shall see, are sufficient to sustain the operability of the system.

ASSUMPTION 3.4: The $F(t)$ input-coefficient matrix is nonnegative (by which Assumption 2.2 is fulfilled), is continuously differentiable, and its spectral radius is less than 1 for every $t \geqslant 0$.

ASSUMPTION 3.5: The x^0 starting production is *continuable*, i.e.,

$$x^0 - F^0x^0 > 0. \tag{3.2}$$

ASSUMPTION 3.6: The $g(t)$ consumption function is *sufficient*,[9] i.e.,

$$g_i(t) > |x_i^0 - \sum_j F_{ij}^0 x_j^0 - g_i^0|(1 + \varepsilon_i) \tag{3.3}$$

[8] Accordingly, we use the following special notation: small letters indicate vectors of n components and capital letters indicate n by n matrices. The components of the vectors, as well as the diagonal entries of the diagonal matrices are marked with a simple subscript; the general entry of a matrix is marked by a double subscript.

Unlike the former convention, we denote by e_j the jth unit vector, and with e the sum of the unit vectors, the summing vector. E_j is the matrix, whose jth diagonal element is 1: all other elements are 0. The sum of these, the unit matrix, is denoted by E.

u:	vector of the producer's stock;
u_i:	the ith sector's stock of its own product;
V:	matrix of the user's stocks;
V_{ij}:	stock at the jth sector of the product of the ith sector;
w:	vector of the consumer's stock;
w_i:	the consumer's stock of the product of the ith sector;
x:	vector of production;
x_i:	quantity produced by the ith sector;
Y:	matrix of the user's purchases;
Y_{ij}:	the jth sector's purchase from the product of the ith sector;
z:	vector of the consumer's purchase;
z_i:	the consumer's purchase from the product of the ith sector;
$g = g(t) > 0$:	the given vector of consumption;
g_i:	consumption from the product of the ith sector;
$F = F(t) \geqslant 0$:	the given matrix of the input coefficients;
F_{ij}:	the used up quantity from the product of the ith sector in the course of making the unit product of the jth sector.

[9] This is a mere technical condition and must not be confused with the economic (or political) requirement that consumption must exceed some subsistence level. This latter requirement must be met when the consumption function is given exogenously.

holds for every $t \geqslant 0$ and for every i, where ε_i is a sufficiently small positive number, independent of t.[10]

ASSUMPTION 3.7: The initial stocks are positive:[11]

$$u^0 > 0, \qquad V^0 > 0, \qquad w^0 > 0. \tag{3.4}$$

We will add some explanation to Assumptions 3.5 and 3.6.

The condition required for the starting production to be continuable is that the net product corresponding to the starting production be positive. This requirement seems almost trivial. In fact, it is not quite so, since an economy may remain able to function for a long time, even if the starting net product has some negative component, because of large enough starting stocks.

The condition that consumption be sufficient (Assumption 3.6) seems more artificial, but is not a very strict one. It requires that the consumption of each product should always exceed (and must not tend to) the absolute value of the difference between the initial net production and initial consumption. The situation is easier to survey in the case discussed in footnote 10, i.e., in the case where $g(t) \geqslant g^0$ for all t and equation (i) (footnote 10) is equivalent to (3.3). The left-hand inequality of (i) can always be satisfied by choosing ε_i sufficiently small. The right-hand inequality requires that initial consumption be slightly higher than half of the initial net production with each product.

3.2. *The Model and its Solution*

Balance equations

Applying the special Assumptions 3.1–3.3, the balance equations take the following forms (the equations are set up in vector form on the left side and in scalar form on the right side):[12]

$$\dot{u} = x - Ye - z, \qquad \dot{u}_i = x_i - \sum_j Y_{ij} - z_i, \tag{3.5}$$

$$\dot{V}e_j = Ye_j - FE_jx, \qquad \dot{V}_{ij} = Y_{ij} - F_{ij}x_j, \tag{3.6}$$

$$\dot{w} = z - g, \qquad \dot{w}_i = z_i - g_i. \tag{3.7}$$

[10] Applying this condition to $t = 0$, we get

$$\left(x_i^0 - \sum_j F_{ij}^0 x_j^0\right)\frac{1 + \varepsilon_i}{\varepsilon_i} > g_i^0 > \left(x_i^0 - \sum_j F_{ij}^0 x_j^0\right)\frac{1 + \varepsilon_i}{2 + \varepsilon_i}. \tag{i}$$

This clearly implies (3.2), thus the latter is redundant. Further, if we suppose that $g(t) \geqslant g^0$ for every t (e.g., $g(t)$ is a monotonically increasing function), then (i) is also both necessary and sufficient for (3.3) to be satisfied.

[11] For a reason that will be clear later, for several conditions that were in weak form ($\geqslant$) in the general model, we require strict inequality ($>$) to hold in the special model.

[12] The turnover balances (2.8)–(2.9) become superfluous in this model.

Rules of behavior

Let us introduce the following constants, which will adjust the control system: u^*, V^*, w^* are the normal stocks of the corresponding variables, and A, C are diagonal matrices of control parameters (A_i, C_i are control parameters of the ith sector).[13]

We apply the following rules of behavior:

$$\begin{aligned} \dot{x} &= \dot{Y}e + \dot{z} - 2AC\dot{u} \\ &\quad + C^2(u^* - u), \end{aligned} \qquad \begin{aligned} \dot{x}_i &= \sum_j \dot{Y}_{ij} + \dot{z}_i - 2A_iC_i\dot{u}_i \\ &\quad + C_i^2(u_i^* - u_i), \end{aligned} \tag{3.8}$$

$$\begin{aligned} \dot{Y}e_j &= FE_j\dot{x} + \dot{F}E_jx - 2AC\dot{V}e_j \\ &\quad + C^2(V^* - V)e_j, \end{aligned} \qquad \begin{aligned} \dot{Y}_{ij} &= F_{ij}\dot{x}_j + \dot{F}_{ij}x_j - 2A_iC_i\dot{V}_{ij} \\ &\quad + C_i^2(V_{ij}^* - V_{ij}), \end{aligned} \tag{3.9}$$

$$\dot{z} = \dot{g} - 2AC\dot{w} + C^2(w^* - w), \qquad \dot{z}_i = \dot{g}_i - 2A_iC_i\dot{w}_i + C_i^2(w_i^* - w_i). \tag{3.10}$$

Solution of the model

The system of linear differential equations (3.5)–(3.10) given above contains $2n^2 + 4n$ scalar equations with the same number of unknown functions. Their coefficient matrix is non-singular; therefore there is only one such solution which, at the point $t = 0$, will take the value u^0, V^0, w^0, x^0, Y^0, z^0. We shall give this solution here explicitly; its correctness can easily be checked by substitution.

But first let us make the assumption that we choose control parameters A_i, C_i so that

$$\begin{aligned} &0 < A_i < 1, \\ &0 < C_i \end{aligned} \tag{3.11}$$

for all i, and let us denote

$$D_i = C_i\sqrt{1 - A_i^2},$$

while D shall be the diagonal matrix formed by the D_is. With these assumptions we have the solution:

[13] A_i turns out to be the damping factor, C_i the natural frequency, A_iC_i the damping exponent, and $C_i\sqrt{1 - A_i^2}$ the damped frequency in the system's transient response.

$$
(3.12)\quad
\begin{aligned}
u &= u^* + e^{-ACt}\{(\cos Dt)(u^0 - u^*) + D^{-1}(\sin Dt) \\
&\qquad \times [x^0 - Y^0 e - z^0 + AC(u^0 - u^*)]\}, \\
Ve_j &= V^* e_j + e^{-ACt}\{(\cos Dt)(V^0 - V^*)e_j + D^{-1}(\sin Dt) \\
&\qquad \times [Y^0 e_j - F^0 E_j x^0 + AC(V^0 - V^*)e_j]\}, \\
w &= w^* + e^{-ACt}\{(\cos Dt)(w^0 - w^*) + D^{-1}(\sin Dt) \\
&\qquad \times [z^0 - g^0 + AC(w^0 - w^*)]\}, \\
x &= (E - F)^{-1} g + (E - F)^{-1} e^{-ACt}\{(\cos Dt)(x^0 - F^0 x^0 - g^0) \\
&\qquad - CD^{-1}(\sin Dt)[A(x^0 - F^0 x^0 - g^0) \\
&\qquad + C(u^0 + V^0 e + w^0 - u^* - V^* e - w^*)]\}, \\
Ye_j &= FE_j x + e^{-ACt}\{(\cos Dt)(Y^0 e_j - F^0 E_j x^0) - CD^{-1}(\sin Dt) \\
&\qquad \times [A(Y^0 e_j - F^0 E_j x^0) + C(V^0 - V^*)e_j]\}, \\
z &= g + e^{-ACt}\{(\cos Dt)(z^0 - g^0) - CD^{-1}(\sin Dt) \\
&\qquad \times [A(z^0 - g^0) + C(w^0 - w^*)]\}.
\end{aligned}
$$

It is apparent that this solution is stable in the sense that all the differences, $u - u^*$, $V - V^*$, $w - w^*$, as well as

$$
(3.13)\quad x - (E - F)^{-1} g, \qquad Ye_j - FE_j(E - F)^{-1} g, \qquad z - g,
$$

tend to 0 as t approaches infinity.

The u, V, and w components of the solution (3.12) are independent of exogeneous consumption g, while the components x, Y, and z are monotonically increasing functions of the same, as could be expected. Taking this fact and the condition of sufficient consumption (3.3) into consideration, it appears that our model may represent either a stagnating or an arbitrarily growing economy. Bounds of growth are determined by the resources neglected in our special model.

3.3. *Functioning Ability of the System*

We define the survival conditions of the system by the following inequalities, similar to the ones admitted in the general model:

$$
(3.14)\quad x > 0,
$$

$$
(3.15)\quad u > 0,
$$

$$
(3.16)\quad V > 0,
$$

$$
(3.17)\quad w > 0.
$$

THEOREM: *In the system fulfilling the general conditions* 2.1–2.4 *and the special conditions* 3.1–3.7 *there exist sets of the control parameters* A_i, C_i *satisfying* *conditions* (3.11) *and sets of the* u_i^*, V_{ij}^*, w_i^* *normal stocks, such that the solution of the system* (3.5)–(3.10) *is able to function in the sense of conditions* (3.14)–(3.17).

PROOF: Set

$$u^* = u^0, \qquad V^* = V^0, \qquad w^* = w^0, \tag{3.18}$$

and consider first condition (3.14) and the expression of x in the solution (3.12). We must have

$$\begin{aligned} x = (E - F)^{-1} e^{-ACt}[e^{ACt}g + (\cos Dt - ACD^{-1} \sin Dt) \\ \times (x^0 - F^0x^0 - g^0)] > 0. \end{aligned} \tag{3.19}$$

$(E - F)^{-1}$ is non-negative, since the spectral radius of F is less than one and so is e^{-ACt}. Thus for the inequality (3.19) to hold it is sufficient that

$$e^{ACt}g + (\cos Dt - ACD^{-1} \sin Dt)(x^0 - F^0x^0 - g^0) > 0$$

or, in terms of components,

$$e^{A_iC_it}g_i + \left(\cos D_it - \frac{A_iC_i}{D_i} \sin D_it\right)\left(x_i^0 - \sum_j F_{ij}^0x_j^0 - g_i^0\right) > 0 \tag{3.20}$$

for $t \geqslant 0$. Since A_i, C_i, and g_i are positive, we have

$$e^{A_iC_it}g_i \geqslant g_i.$$

Thus for (3.20) it is sufficient if

$$g_i(t) > -\left(\cos D_it - \frac{A_iC_i}{D_i} \sin D_it\right)\left(x_i^0 - \sum_j F_{ij}^0x_j - g_i^0\right).$$

Taking the amplitude of the right-hand periodic function, we get

$$g_i(t) > \left(1 + \frac{A_i^2C_i^2}{D_i^2}\right)^{1/2}\left|x_i^0 - \sum_j F_{ij}^0x_j^0 - g_i^0\right| = \frac{|x_i^0 - \sum_j F_{ij}^0x_i^0 - g_i^0|}{\sqrt{1 - A_i^2}}. \tag{3.21}$$

If, for some i, $|x_i^0 - \Sigma_j F_{ij}^0x_j^0 - g_i^0|$ happens to be 0, then (3.21) holds for arbitrary A_i by (3.3). If $|x_i^0 - \Sigma_j F_{ij}^0x_j^0 - g_i^0| > 0$, then

$$1 + \varepsilon_i \geqslant \frac{1}{\sqrt{1 - A_i^2}}$$

is sufficient for (3.21) to hold. Therefore we choose[14]

$$A_i \leqslant \frac{\sqrt{2\varepsilon_i + \varepsilon_i^2}}{1 + \varepsilon_i}, \tag{3.22}$$

[14] It is expedient to choose A_i at or near the upper bound in order to speed up damping.

whereby conditions (3.14), positivity of the production vector x, is accomplished. Taking the choice (3.18) of the normal stocks into account, conditions (3.15)–(3.17) require

$$\begin{aligned} & e^{ACt}u^0 + D^{-1}\sin Dt(x^0 - Y^0e - z^0) > 0, \\ (3.23) \qquad & e^{ACt}V^0e_j + D^{-1}\sin Dt(Y^0e_j - F^0E_jx^0) > 0, \\ & e^{ACt}w^0 + D^{-1}\sin Dt(z^0 - g^0) > 0. \end{aligned}$$

A similar but simpler reasoning than that applied to derive (3.22) from (3.19) yields the following sufficient condition for (3.23):

$$(3.24) \qquad C_i > \frac{1}{\sqrt{1 - A_i^2}} \max\left\{ \frac{|x_i^0 - \sum_j Y_{ij}^0 - z_i^0|}{u_i^0}; \max_j \frac{|Y_{ij} - F_{ij}^0 x_j^0|}{V_{ij}^0}; \frac{|z_i^0 - g_i^0|}{w_i^0} \right\}$$

where A_i has been chosen according to (3.22).

We have thus proved the theorem and, under (3.18), (3.22), and (3.24) we have also specified a possible choice of the normal stocks and the control parameters. It must be remarked, however, that the inequality system (3.14)–(3.17) can be satisfied even if (3.18) is not strictly fulfilled, but the conditions for this would be very difficult to produce in explicit form.

3.4. *Perturbation of the System*

The system described above can be rightly called controlled only if it is also able to function in the presence of some external disturbances. To investigate the problem, we submit the system to two different kinds of perturbations. In the course of these investigations we assume that the unperturbed system is able to function, i.e., we chose the normal stocks and the control parameters in such a way that the solution of the unperturbed system should meet the inequalities (3.14)–(3.17). On the other hand, we have not assumed that these conditions are fulfilled exactly in the way given in the relations (3.18), (3.22), and (3.24).

First perturbation: Deviation from the initial values

Let us assume that the initial values u^0, V^0, w^0, x^0, Y^0, and z^0 were settled inaccurately at the introduction of the system and that the initial values were really $\tilde{u}^0, \tilde{V}^0, \ldots, \tilde{z}^0$, such that in the $2n^2 + 4n$ dimensional space of the variables $u, V, \ldots, z$ the point $P = (u^0, V^0, \ldots, z^0)$ lies suitably near to the point $\tilde{P} = (\tilde{u}^0, \tilde{V}^0, \ldots, \tilde{z}^0)$.

To the two different starting points correspond two different paths (represented by the equations of motion, (3.5)–(3.10)). The difference of the co-ordinates will be written down only for the first $n^2 + 3n$ co-ordinates, i.e., for those which are prescribed by (3.14)–(3.17) to be positive:

$$
\begin{aligned}
\tilde{u} - u &= e^{-ACt}\{(\cos Dt)(\tilde{u}^0 - u^0) + D^{-1}(\sin Dt) \\
&\qquad \times [\tilde{x}^0 - \tilde{Y}^0 e - \tilde{z}^0 - x^0 + Y^0 e + z^0 + AC(\tilde{u}^0 - u^0)]\}, \\
(\tilde{V} - V)e_j &= e^{-ACt}(\cos Dt)(\tilde{V}^0 - V^0)e_j + D^{-1}(\sin Dt)[(\tilde{Y}^0 - Y^0)e_j \\
&\qquad - F^0 E_j(\tilde{x}^0 - x^0) + AC(\tilde{V}^0 - V^0)e_j]\}, \\
\tilde{w} - w &= e^{-ACt}\{(\cos Dt)(\tilde{w}^0 - w^0) + D^{-1}(\sin Dt)[\tilde{z}^0 - z^0 + AC \\
&\qquad \times (\tilde{w}^0 - w^0)]\}, \\
\tilde{x} - x &= (E - F)^{-1} e^{-ACt}\{(\cos Dt)(E - F^0)(\tilde{x}^0 - x^0) - CD^{-1} \\
&\qquad \times (\sin Dt)[A(E - F^0)(\tilde{x}^0 - x^0) + C(\tilde{u}^0 + \tilde{V}^0 e \\
&\qquad + \tilde{w}^0 - u^0 - V^0 e - w^0)]\}.
\end{aligned}
\tag{3.25}
$$

One sees that the absolute value of each component of the expression on the right side of (3.25) will be less than any $\varepsilon > 0$ number, independent of time, if P lies near enough to $\tilde{P}$, for every $t \geqslant 0$. Consequently, the absolute value of the difference between the co-ordinates will be less than this ε for every t. Since the above-mentioned co-ordinates of the unperturbed solution are positive, by choosing ε sufficiently small, those of the perturbed solution will also be positive.

Second perturbation: Sudden change of stocks

Here we assume that at the moment $t = \tilde{t}$ the stocks, instead of the values given by the solution (3.12) suddenly take different values $\tilde{u}$, $\tilde{V}$, and $\tilde{w}$. This of course implies that the balance equations (3.5)–(3.7) are not valid at the moment $\tilde{t}$ (they even lack interpretation). On the other hand we assume that in other respects the economy functions further as if nothing had happened, in accordance with the rules of behavior (3.8)–(3.10).

We will not present the whole tedious discussion of this perturbation, but rather a heuristic reasoning for $\tilde{t}$ sufficiently large. This is to say that we suppose that at $\tilde{t}$ the system is already in steady state, or in technical terms:

$$
\begin{aligned}
x(\tilde{t}) &= [E - F(\tilde{t})]^{-1} g(\tilde{t}), \\
Y(\tilde{t})e_j &= F(t)E_j[E - F(\tilde{t})]^{-1} g(\tilde{t}), \\
z(\tilde{t}) &= g(\tilde{t}).
\end{aligned}
\tag{3.26}
$$

The perturbation may be interpreted as if the functioning of the system started anew at time $t = \tilde{t}$, with initial stocks $\tilde{u}$, $\tilde{V}$, and $\tilde{w}$, and under conditions (3.26).

For the sake of simplicity we shift the time scale so that $\tilde{t} = 0$, and then we have from (3.26)

$$
\begin{aligned}
&x^0 - F^0x^0 - g^0 = 0,\\
(3.27)\qquad &Y^0e_j - F^0E_jx^0 = 0,\\
&z^0 - g^0 = 0,
\end{aligned}
$$

and also

$$
(3.28)\qquad x^0 - Y^0e - z^0 = 0.
$$

Therefore after changing the stocks at $\tilde{t}$, the solution (3.13) takes the following form:

$$
\begin{aligned}
&u = u^* + e^{-ACt}(\cos Dt + ACD^{-1}\sin Dt)(\tilde{u} - u^*),\\
(3.29)\qquad &Ve_j = V^*e_j + e^{-ACt}(\cos Dt + ACD^{-1}\sin Dt)(\tilde{V} - V^*)e_j,\\
&w = w^* + e^{-ACt}(\cos Dt + ACD^{-1}\sin Dt)(\tilde{w} - w^*),\\
&x = (E - F)^{-1}[g - e^{-ACt}C^2D^{-1}\sin Dt(\tilde{u} + \tilde{V}e + \tilde{w}\\
&\qquad - u^* - V^*e - w^*)].
\end{aligned}
$$

First we observe that by the first equation of (3.27) both conditions (3.2) and (3.3) are satisfied in the system starting at $\tilde{t}$.

The positivity of the variables u, V, w, and x will depend on the size of the jump. Testing u first we must have

$$
u^*\, e^{ACt} > (\cos Dt + ACD^{-1}\sin Dt)(\tilde{u} - u^*)
$$

or, as a sufficient condition in terms of components:

$$
u_i^* > \left(1 + \frac{A_i^2C_i^2}{D_i^2}\right)^{1/2}|\tilde{u}_i - u_i^*| = \frac{|\tilde{u}_i - u_i^*|}{\sqrt{1 - A_i^2}}.
$$

This is

$$
(3.30)\qquad (1 + \sqrt{1 - A_i^2})u_i^* > \tilde{u}_i > (1 - \sqrt{1 - A_i^2})u_i^*.
$$

Considering that A_i is small, the value $\sqrt{1 - A_i^2} \cong 1$ and the permissible range of $\bar{u}_i$ is approximately twice the normal stock.

Similar estimation can be made in the permitted range of $\tilde{V}$ and $\tilde{w}$, and the conclusion can be drawn that at a point of time that is sufficiently far from the starting of the system an abrupt change in the individual stocks (whether in a positive or negative direction) can be nearly as large as the normal stock itself without driving the corresponding stock to a negative value in the future and without a need for readjusting the control parameters.

The required conditions, however, for production to remain positive also are more restrictive. Though a somewhat looser estimate can easily be made, we apply to x reasoning analogous to that applied to u, and obtain the following sufficient condition:

$$g_i(t) > \frac{C_i}{\sqrt{1 - A_i^2}} \left| \tilde{u}_i + \sum_j \tilde{V}_{ij} + \tilde{w}_i - u_i^* - \sum_j V_{ij}^* - w_i^* \right| \tag{3.31}$$

for all i and $t \geqslant \tilde{t}$. Inequality (3.31) shows that a pure (irregular) transfer from one stock to the other will not impede the operation of the system. But an uncompensated deviation in any individual stock from the normal stock must be within a possibly small range, particularly for a product whose consumption is not large enough. This range also depends heavily on C_i and may be a large number. In (3.24) we had only a lower bound on C_i and it seemed that a large C_i was favorable because of its damping effect. Now we have also observed its drawback: the larger the C_i, the more sensitive is the system to abrupt changes in the stocks, and control must often be readjusted. Therefore a reasonable choice is to be made, supposedly not too far from the lower bound.

4. CONCLUSION AND GENERAL REMARKS

4.1. *Further Research on Autonomous Control*

The simple model described in Section 3 is a member of the "model family" outlined in Section 2. Its elaboration is considered only the first step in the theoretical investigation of the problem. Further members of the same model family may be formed according to points of view as follows: (i) rendering the model stochastic; (ii) introduction of non-linear production functions; (iii) explicit consideration of limited resources; (iv) introducing technological choice and the substitution among products into the model; (v) employing other rules of behavior; and (vi) taking the time lag of control into consideration.

In Section 3 we could prove that, assuming strong restrictions, autonomous control is able in itself to make the real sphere function and, more than a mere stagnation or "vegetation," may even ensure a certain progress. We are investigating the "limits" to this statement by weakening the restrictions successively. To which of the control functions is the autonomous mechanism able to attend, and to which is it not?

We shall have to answer questions of the following kind: Can the real sphere directed by autonomous control adapt itself only to small perturbations, not affecting too greatly the original balance position, or can it also adapt to larger ones? Can the real sphere be autonomously controlled even if there are external limited resources, especially if the volume and proportions of the external resources change in time as, for example, if one resource grows scarcer compared to another? The following question is related to the previous one. Can the real sphere be autonomously controlled only in case of a given technology, or is autonomous control able to also direct the technological change?

And finally, how efficient, wasteful, or economical is autonomous control? Its drawbacks are securing stocks and the losses entailed in the fluctuation of production and purchase. On the other hand, it has the advantage that it gets information very inexpensively and its administrative costs are low.

In further theoretical research we can make use of the results of control theory. This theory is well built up in its conceptual system, which has been used mainly for technical purposes, but which is able to describe controlling processes abstractly and to analyze the characteristics of these independent of the composition of the controlled real process. H. A. Simon was the pioneer of the application of control theory in economics (microeconomics), and his ideas have many features in common with ours (see [7]). It seems that his initiative was not much followed. (The theory of optimal control is farther from the field of our research; therefore we do not deal with it here.)

Further important branches of science are the operations research models and theorems worked out for the purposes of optimal inventory. A number of theoretical economists have dealt with the latter subject, but always from the microeconomic point of view (see [**2**, **3**]). Although in many cases there was a veritable personal union among the economists researching on the one hand the *whole* of the economic system (e.g., the general theory of equilibrium), and on the other hand the *partial* problem, i.e., the firms' inventory problem, these two spheres of the theory have never really been integrated. This integration has now become timely. The stock has many functions. One of its duties (which was the subject of operational research) is to render the work of the firm and the service to the firm's customers undisturbed. Its other function is that which the present study points out, i.e., its service as a signalling system for the control of the whole economy.

4.2. *Complex, Multiple-Stage Control*

We consider the study of "pure" control mechanisms as preparation for the actual, important scientific task: the working out of the theory of *complex* control mechanisms.

One of the main objects of our study is the dethronement of the "pure" price mechanism. We want to demonstrate that there exists a mechanism functioning *without* price signals which, under abstract conditions, is able to make the real sphere of an economy operate and to assure the survival of the system. By this we do not seek to put control based on stock signals on the throne. A pure price mechanism,[15] or a mechanism based purely on stock signals can in themselves effect control of the real sphere only under strongly simplified assumptions in the paper world of an abstract model. In a real economy no "pure" mechanism can in itself assure survival of the system, its effective adaptation, and harmonic growth.

The control of the actual economy is complex and multiple-stage. To start with, there is a *division of work* among the different control mechanisms; they complete each other. In a modern economy, pricing is in many markets rigid and inefficient.

[15] Since the volume of this paper is restricted, we cannot undertake to contrast the assumptions of theories concerned with the "pure" price mechanism with economic reality here. J. Kornai gave detailed comments criticizing neo-classical price theory and general equilibrium theory in his work [**4**].

Through autonomous control the system can easily adjust itself to minor changes before these are indicated either by prices or by central instructions. The importance of autonomous control is explained (in our view) by the fact that this mechanism is simple, even primitive, therefore quick and inexpensive. The simple mechanism of autonomous control cannot direct a complicated task of adaptation, let alone the preliminary adaptation to future changes.

There is not only a division of work among the controlling mechanisms, but also an *overlap*. The same real process is controlled by the different mechanisms *jointly*. None of the mechanisms is fully reliable in itself. If they are suitably connected, they can, by their joint effect, mutually correct each other's mistakes, subdue oscillations, and reduce adaptational losses. (Of course, in case of a wrong connection they may also mutually strengthen the oscillations and the losses.)

The most important thing to be considered in this respect is experience. The control of every modern, actual economy is complex and multi-stage. It is therefore likely such a control is needed inevitably. It is not worth the attempt to create an optimal, "pure" mechanism (e.g., an optimal price system) that would by itself accomplish all control. Examining the roles of prices of short-term instructions, of stock signals, or of the medium and long term plans, in the end we shall have to pose the question: how do these sources of information *jointly* affect the real processes of the economy?

This is a question which our science has not yet really asked itself. We cannot expect a quick answer to it either. We already have a number of models of one of the controlling mechanisms, the price mechanism. In the following phase of our research work we need to model the other mechanisms (directive control, planning, autonomous control, etc.). This may be followed by a task seemingly even more difficult, but very important to solve: the theoretical analysis of the multi-stage, complex system of control.

Institute of Economics, Hungarian Academy of Sciences

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